From surface-enhanced field penetration to robust quantum fractality in artificially curved spacetime:

Topological superconductors and surface Majorana fluids

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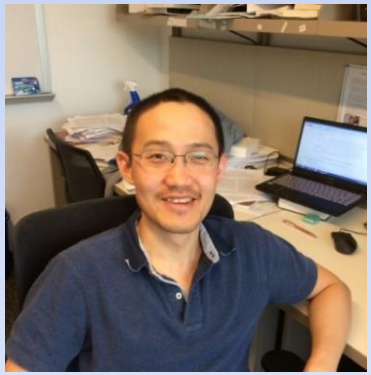

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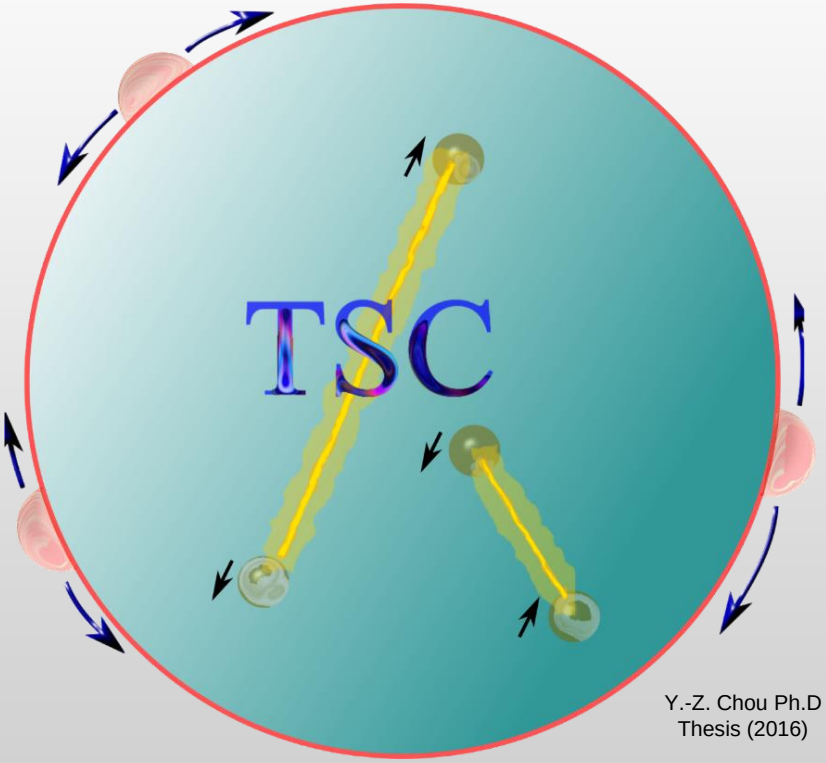
3D Topological Superconductivity

- 3D gapped bulk, integer-valued winding number
- 2D gapless *surface fluid* of Majorana fermion quasiparticles



Y.-Z. Chou Ph.D Thesis (2016)

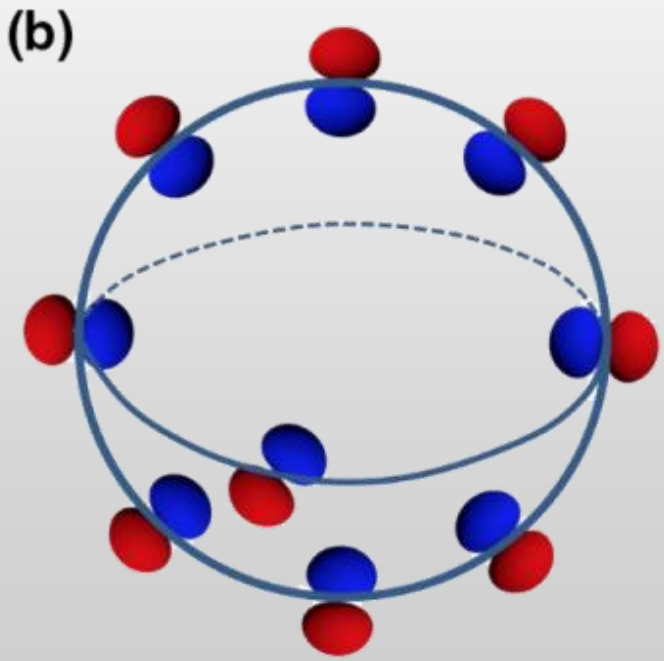
Schnyder, Ryu, Furusaki, Ludwig (2008)
R. Roy (2008); A. Kitaev (2009); G. Volovik (2009)
Qi, Hughes, Raghu, Zhang (2009)

3D Topological Superconductivity

- 3D gapped bulk, integer-valued winding number
- 2D gapless *surface fluid* of Majorana fermion quasiparticles
- Topological bulk pairing:

E.g., isotropic p-wave $\Delta \hat{\vec{S}} \cdot \vec{p}$

(as in ^{3}He-*B*, class DIII)



Yang, Li, and Wu (2016)

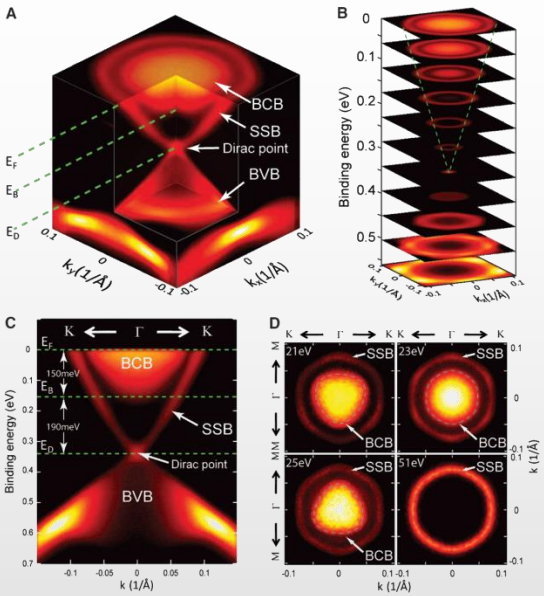
G. Volovik, *The Universe in a Helium Dropet* (2003)
Schnyder, Ryu, Furusaki, Ludwig (2008)
R. Roy (2008); G. Volovik (2009)
Qi, Hughes, Raghu, Zhang (2009)
M. Sato (2009,2010); Fu and Berg (2010)

1. Solid state TSCs: Pairing in Tis, semimetals with SOC?

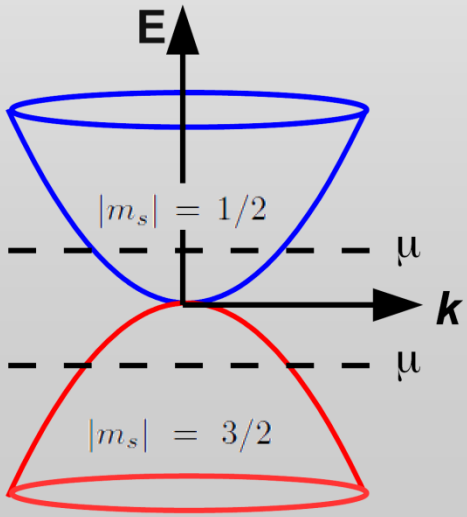
- **Where to look for TSCs? One idea:**
- Semimetals, narrow-gap semiconductors, TIs with strong spin-orbit coupling (SOC)

Why?

1. Small doping, low-carrier density:
   Strong interactions (Coulomb suppresses s-wave)
2. Large renormalization from high density:
   Local pairing operators dominate
3. BCS pairing in conduction/valence band:
   Projected local pairing operators: p-,d-wave…



Y. L. Chen et al. (2010)



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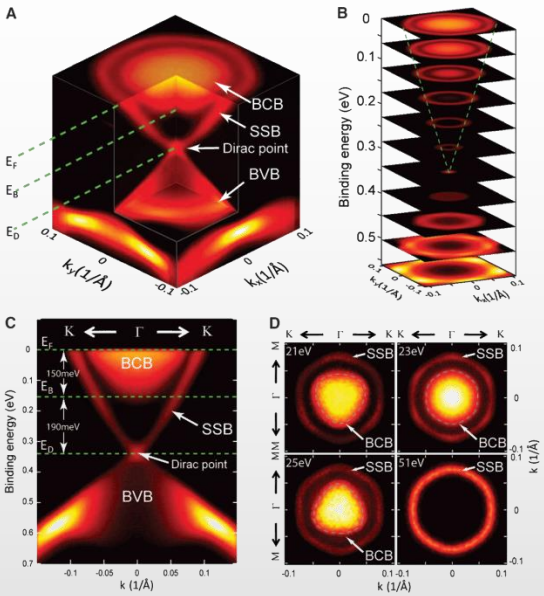


Y. L. Chen et al. (2010)

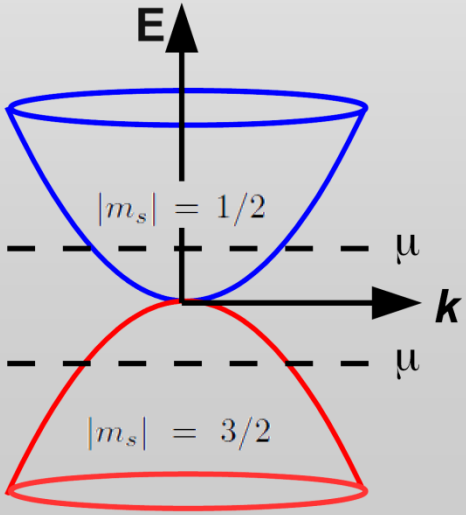
Examples:

- **Dirac semimetal or TI:** 4 of 6 projected local pairing operators are odd-parity, p-wave, topological

  M. Sato (2009,2010); Fu and Berg (2010); L. Fu (2014)

- **Luttinger semimetal:** 5 of 6 projected local pairings are even-parity, d-wave (e.g., d+id Weyl)

  Brydon, Wang, Weinert, Agterberg (2016); Savary, Ruhman, Venderbos, Fu, Lee (2017); Roy, Ghorashi, Foster Nevidomskyy (2019); Sim, Mishra, Park, Kim, Cho, Lee (2018)



1. Solid state TSCs: Pairing in Tis, semimetals with SOC?

Not the whole story, though

E.g., Luttinger semimetal can host odd-parity states:

- "Septet," orbital p-wave pairing with nodal loops

  Brydon, Wang, Weinert, Agterberg (2016); H. Kim *et al.* (2018)

- ^{3}He-*B* $\Delta \hat{\vec{S}} \cdot \vec{p}$ pairing might be favored by electron-optical phonon interactions in half-Heusler compounds

  Savary, Ruhman, Venderbos, Fu, Lee (2017); see also Roy, Ghorashi, Foster Nevidomskyy (2019)

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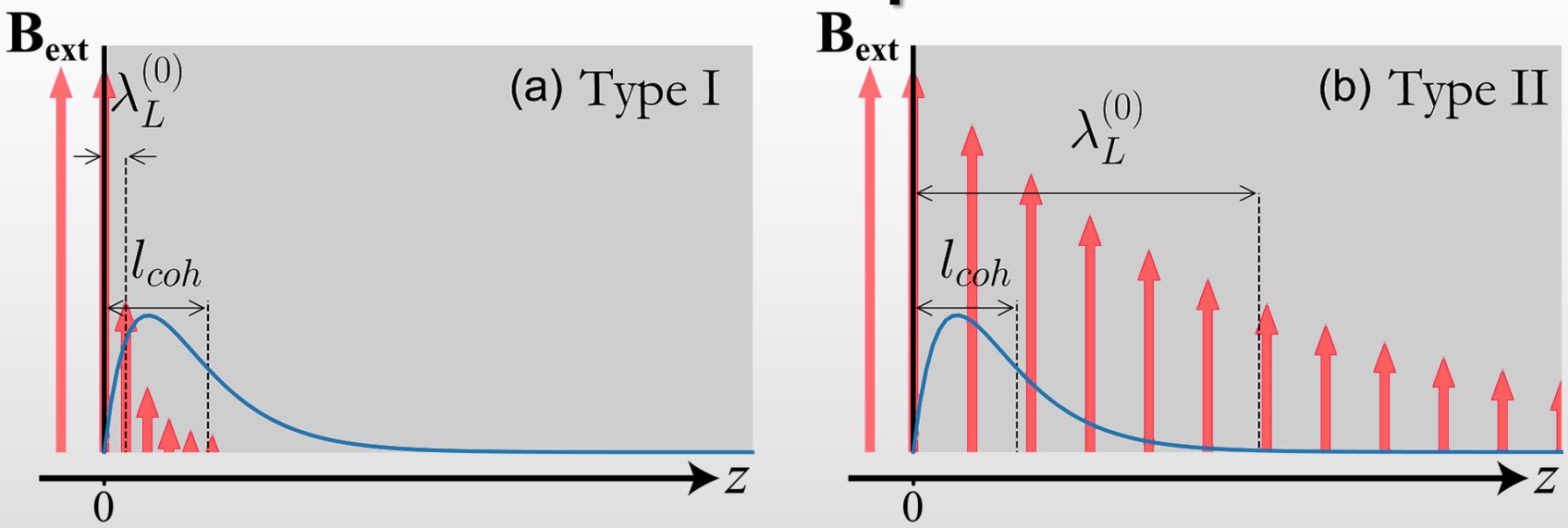
Real materials:

1. $Cu_xBi_2Se_3$
   - Lots of evidence for **nematicity** (**NMR** K. Matano *et al.* 2016, **specific heat** S. Yonezawa *et al.* 2017, **STM** R. Tao *et al.* 2018; see also Chen, Chen, Yang, Du, Wen 2018)
   - No conclusive evidence for **Majorana surface fluid** (**SPCS** S. Sasaki *et al.* 2011, Peng, De, Lv, Wei, Chu 2013; **STM** N. Levy *et al.* 2013; R. Tau *et al.* 2018)
   - Very small fraction (5%) of surface shows signatures of superconductivity in STM! (R. Tao *et al.* 2018)

2. $Nb_xBi_2Se_3$
   - Evidence for nematicity (**Torque magnetometry** T. Asaba et al. 2017)
   - Power-law temperature-suppression in penetration depth (M. P. Smylie *et al.* 2016, 2017)

3. YPtBi
   - Power-law temperature-suppression in penetration depth (H. Kim *et al.* 2018)

Meissner field penetration



- Red arrows: magnetic field
- Blue curves: Majorana surface density profile

Length scales:

1. London (bulk diamagnetic) field penetration depth $\lambda_L^{(0)}$
2. Coherence length = surface fluid depth l_{coh}

Linear Response Theory

$$\nabla^2 \mathbf{A} = -\frac{4\pi}{c}\mathbf{J}$$

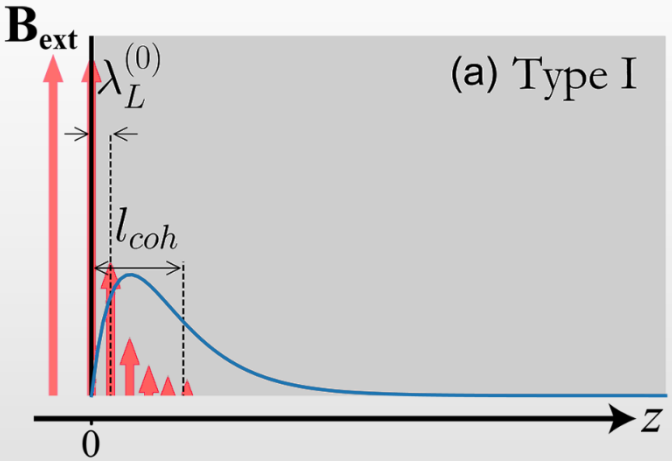
$$-2B_y^{(0)} = \underbrace{\left[q_z^2 + \left(\lambda_L^{(0)}\right)^{-2}\right]}_{\text{London response}} \tilde{A}(q_z) + \frac{4\pi}{c^2}\int_{Q_z} \underbrace{\tilde{\Pi}^{xx}_{1,R}(0,0,0;q_z,-Q_z)}_{\text{Paramagnetic current-current}} \tilde{A}(Q_z)$$

- Calculate paramagnetic current-current correlator for TSC, invert integral equation to get $A(z), \quad B(z) = \partial_z A(z)$
- Bulk-bulk (fully gapped!) paramagnetic response exponentially suppressed at low $T \ll T_c$
- *Surface-surface contribution?*

Model: "Solid-state ^{3}He-*B*"

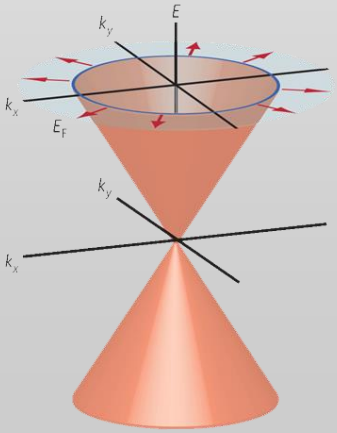
$$H = \frac{1}{2}\int \frac{d^2\mathbf{k}\, dk_z}{(2\pi)^3}\, \chi^\dagger\, \hat{h}\, \chi,$$

$$\hat{h}(\mathbf{k}, k_z) = \left(\frac{k^2+k_z^2}{2m} - \mu\right)\hat{\sigma}^3 + \Delta\left(\hat{\mathbf{s}}\cdot\mathbf{k} + \hat{s}^3 k_z\right)\hat{\sigma}^2$$



- Hard wall boundary condition at $z = 0$
- Surface states with $\varepsilon_{\mathbf{k}} = \Delta|\mathbf{k}|$:

$$\psi^{\mathsf{s}}_{\mathbf{k}}(z) = \frac{e^{-z/l_{\mathsf{coh}}}}{\sqrt{N^{\mathsf{s}}_{\mathbf{k}}}} \sin\left(z\sqrt{k_F^2 - l_{\mathsf{coh}}^{-2} - k^2}\right)|\psi^0_{\mathbf{k}}\rangle$$



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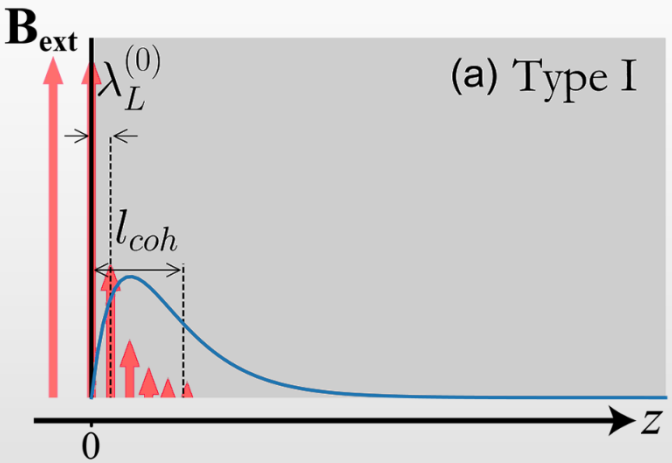


- Surface-surface current-current correlator:

$$\tilde{\Pi}^{xx}_{1,R,\mathsf{s},\mathsf{s}}(0,0,0;q_z,-Q_z) \simeq -\mathcal{C}\left(\frac{e}{m}\right)^2 \frac{(k_B T)^3}{\Delta^4}\,\Theta(0,q_z)\,\Theta(0,-Q_z),$$

$$\Theta(k,q_z) \equiv \int_0^\infty dz\, e^{-iq_z z}\, \psi^{\mathsf{s}\dagger}_{\mathbf{k}}(z)\, \psi^{\mathsf{s}}_{\mathbf{k}}(z)$$

- Two powers of *T* from current operator, one from surface DoS

Main Results

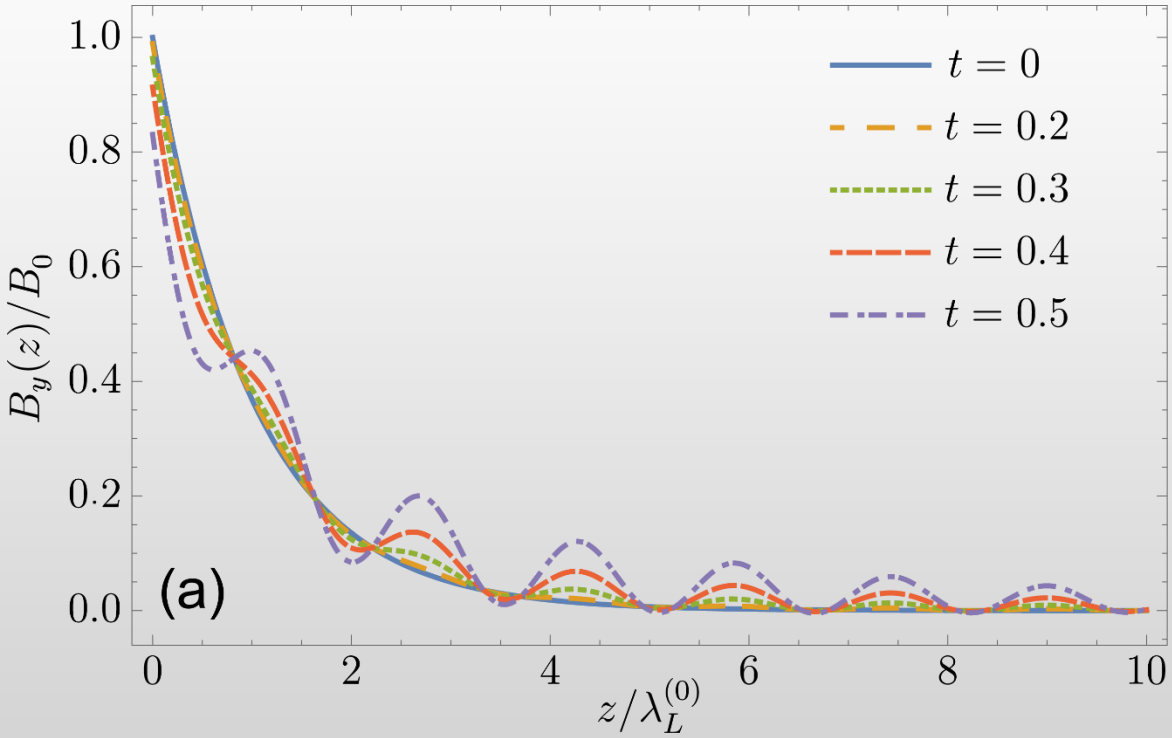
$$B_y(z) = B_y^{(0)}\, \mathrm{e}^{-z/\lambda_L^{(0)}} - B_y^{(0)}\, \varrho(T) \Big[\partial_z G(z)\Big] G(0)$$

London response Correction due to the surface fluid

- Solve integral equation for field
- Length scale $\varrho(T) = |\#| \left(\lambda_L^{(0)}\right)^6 l_{\mathrm{coh}}^{-1}\, k_F^4\, t^3, \quad t \equiv \dfrac{k_B T}{\Delta k_F}$
- Dimensionless function *G*(*z*) from convolution of London, surface responses
- Change in penetration depth $\delta\lambda = \varrho(T) G(0) G(0) > 0$ $\propto T^3$

Wu, Pal, Hosur, Foster (to appear)

Type I $\left(\lambda_L^{(0)} \ll l_{\text{coh}}\right)$ field profile



$$t = \frac{k_B T}{\Delta k_F}$$

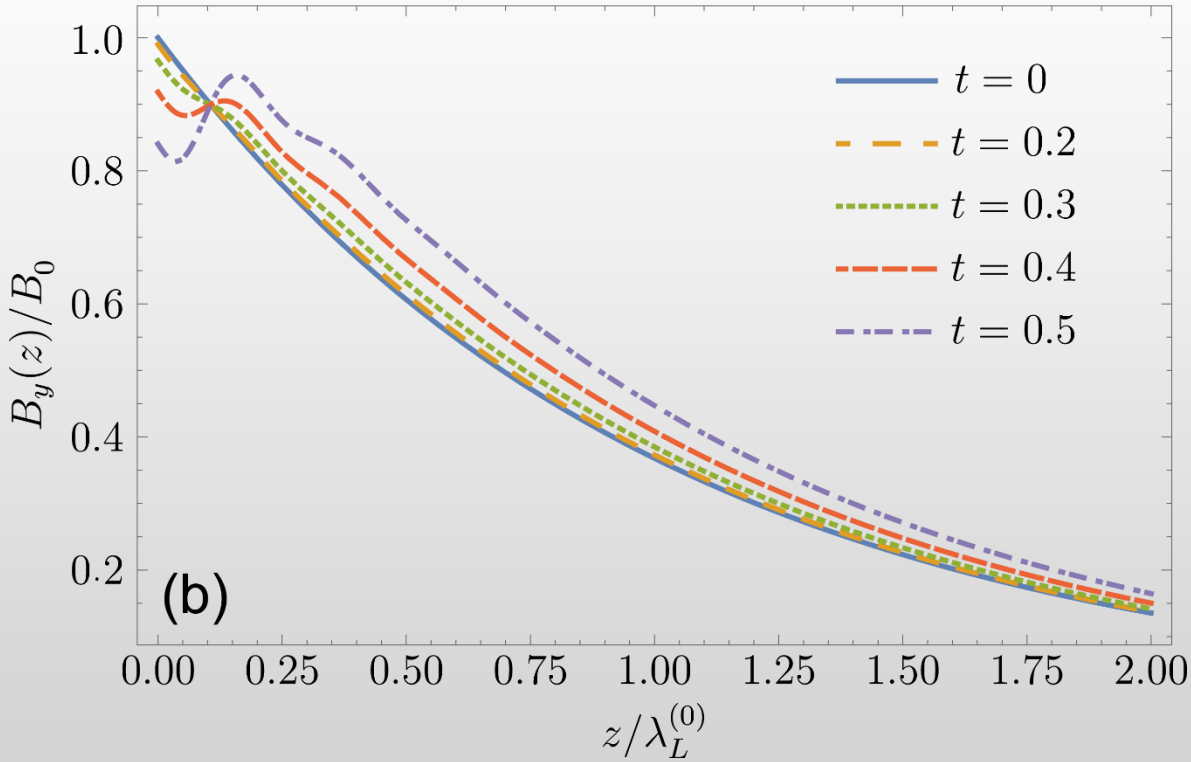
$$k_F^{-1} = 0.5\,\lambda_L^{(0)}$$

$$l_{\text{coh}} = 10\,\lambda_L^{(0)}$$

- Deeper penetration due to surface fluid
- Friedel oscillations in profile

Wu, Pal, Hosur, Foster (to appear)

Type II $\left(\lambda_L^{(0)} \gg l_{\text{coh}}\right)$ field profile



$$t = \frac{k_B T}{\Delta k_F}$$

$$k_F^{-1} = 0.07\,\lambda_L^{(0)}$$

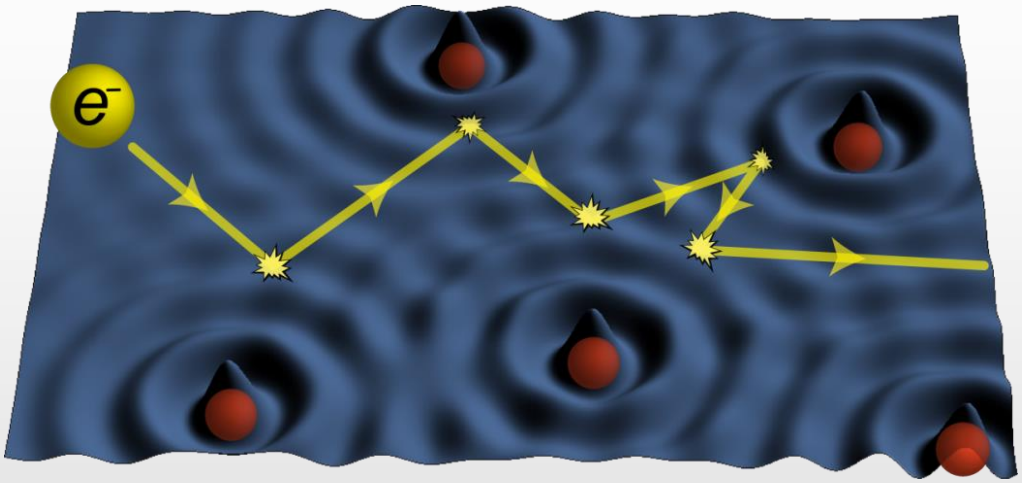
$$l_{\text{coh}} = 0.27\,\lambda_L^{(0)}$$

- Strongest modulation near the surface
- Field penetration governed by London, *enhanced ampitude*

Wu, Pal, Hosur, Foster (to appear)

3. TSCs and Quantum corrections to surface transport

Interaction corrections to transport: Altshuler-Aronov



In a disordered, interacting electron system, electrons multiply scatter elastically off of

- **Impurities (weak localization, etc)**
- **$2k_F$ Friedel oscillations in the density *induced by the impurities***

➡ **Coherent scattering for low-energy carriers near the Fermi energy!**

Altshuler and Aronov 1985 (Review)
Aleiner, Altshuler, and Gershenson 1999 (Review)
Zala, Narozhny, and Aleiner 2001

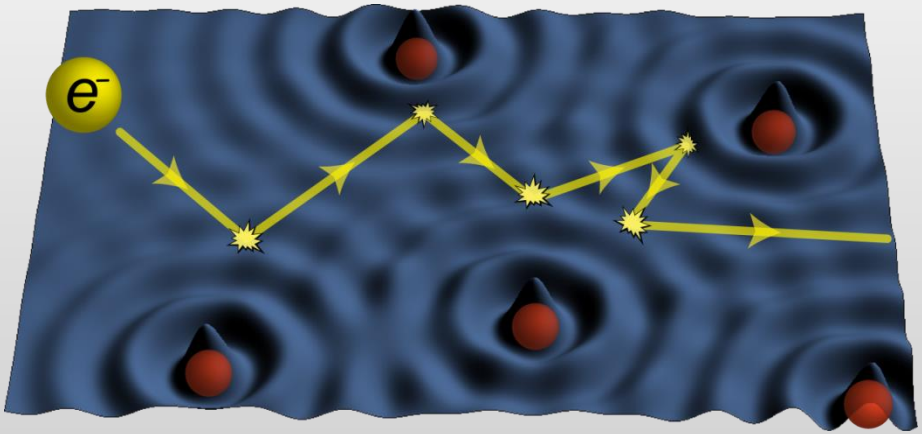
3. TSCs and Quantum corrections to surface transport

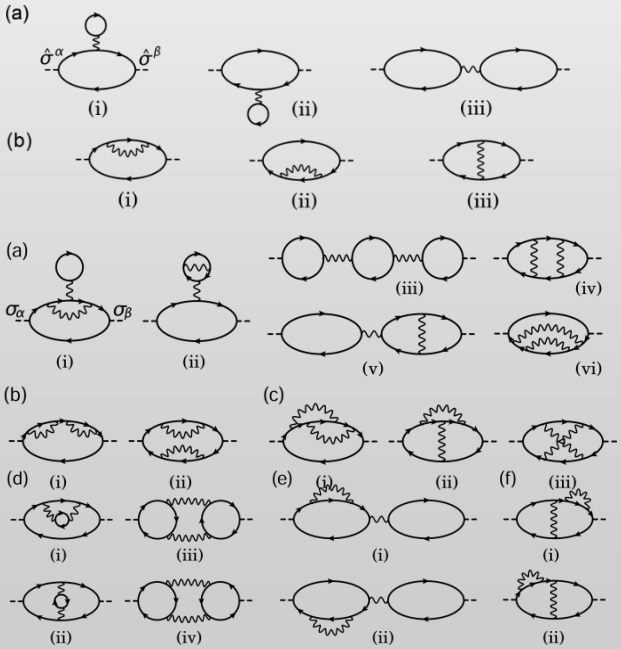
Surface Majorana fluid can carry spin or heat

Quantized surface transport coefficients at zero temperature, without interactions.

Ludwig, Fisher, Shankar, Grinstein (1994)
A. Tsvelik (1995)
Ostrovsky, Gornyi, Mirlin (2006)

Scattering off of impurity-induced density Friedel oscilations (Altshuler-Aronov corrections, short-ranged interactions)





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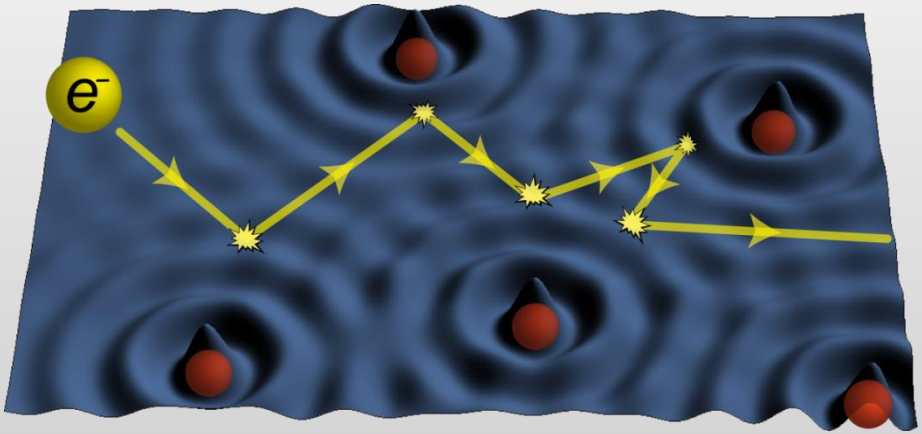

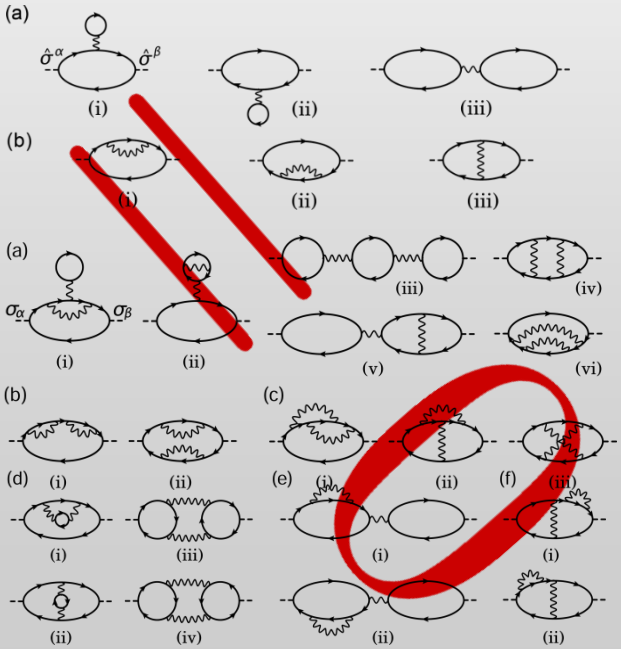

Vanish in every fixed disorder realization.

Ingredients:

1. **Anomalous time-reversal symmetry (topology!)**
2. **Spin U(1) Ward identity**

Xie, Chou, and Foster (2015)

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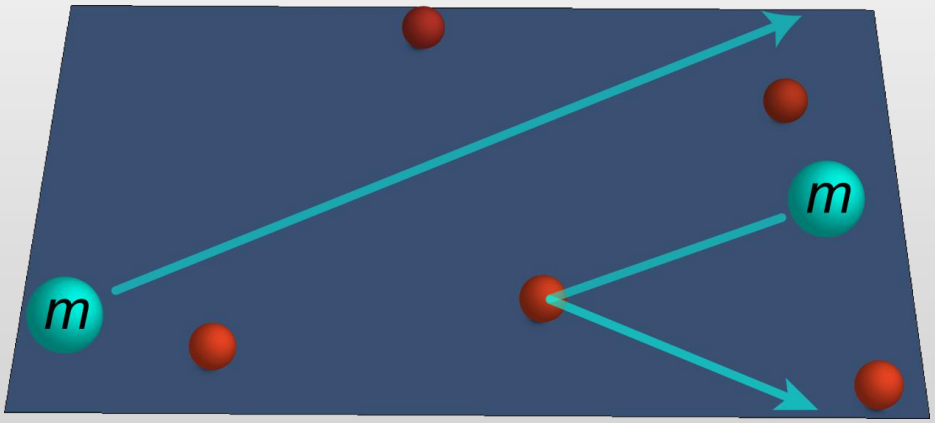


Anomalous time-reversal symmetry:

No Majorana "density" (mass, spin, color) can ripple (or become non-zero).

Large winding number expansion confirms.

Universal spin, thermal conductivities!

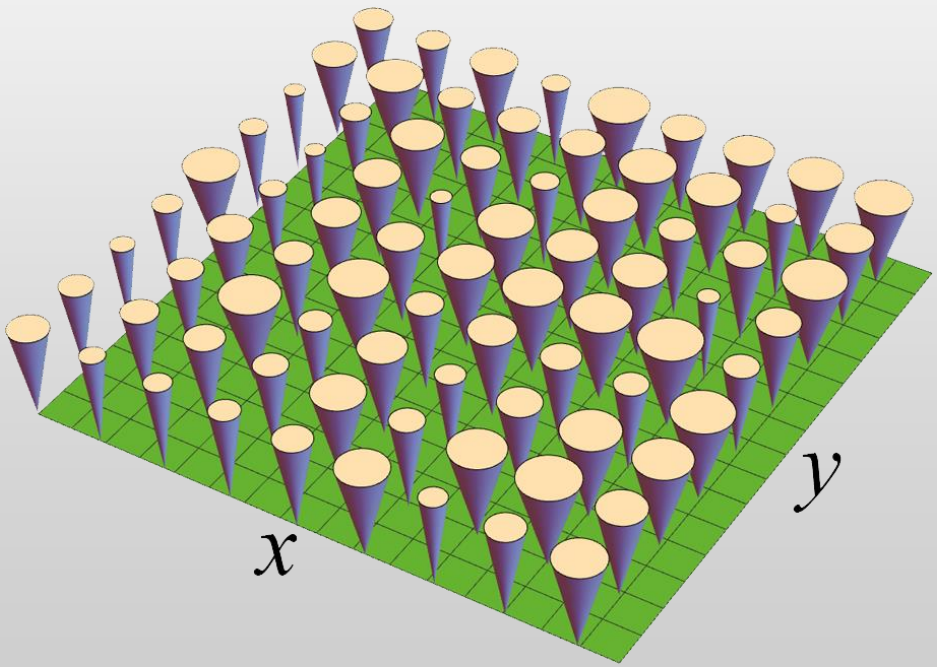
$$\sigma_s = \frac{|\nu|}{\pi h}\left(\frac{\hbar}{2}\right)^2$$

$$\frac{\kappa}{T} = \frac{|\nu|}{\pi h}\frac{\pi^2 k_B^2}{6}$$

Xie, Chou, and Foster (2015)

4. "Artifical gravity" and robust fractality

What happens if we spatially modulate the "speed of light" in a 2D Dirac material?



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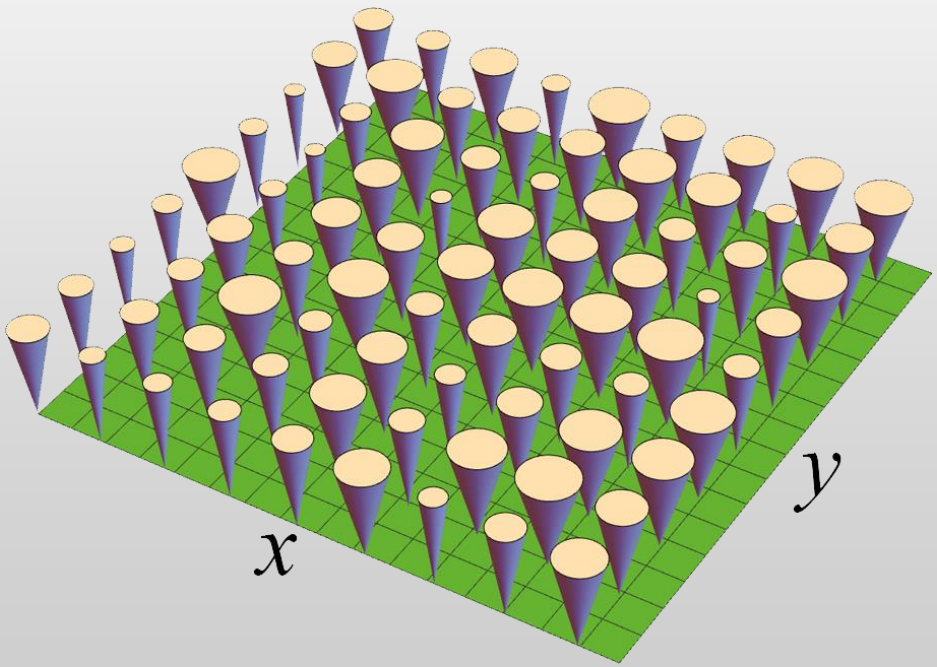

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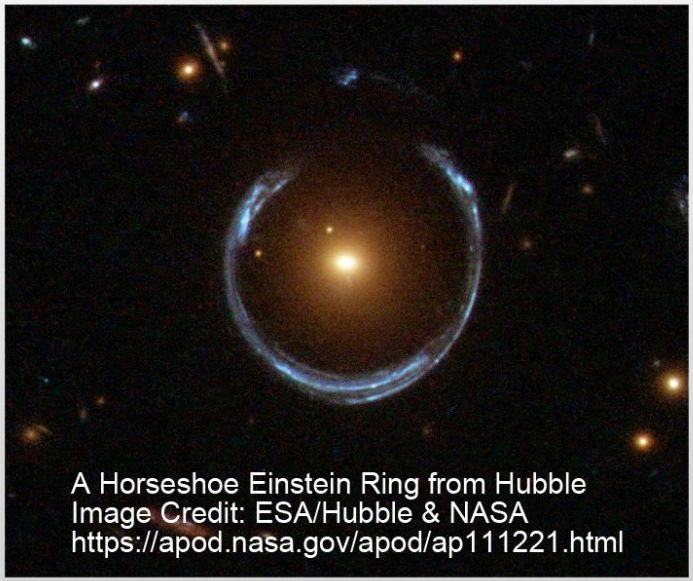

"Gravitational" lensing of relativistic carriers

2+1-D Dirac fermion propagating through curved spacetime:

$$S = \int \sqrt{|g|}\, d^3x\, \bar{\psi}\, E^{\mu}_{A}\, \hat{\gamma}^{A} \left(i\partial_\mu - \frac{1}{2}\omega_\mu{}^{BC}\, \hat{S}_{BC} \right) \psi$$

Spin connection

Spacetime volume measure

Dirac matrices

$$\{\hat{\gamma}^A, \hat{\gamma}^B\} = 2\eta^{AB}\, \hat{1}$$

"Dreibein":

$$g^{\mu\nu} = E^{\mu}_{A}\, E^{\nu}_{B}\, \eta^{AB}$$

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Specialize to a static, spatially inhomogeneous velocity of light $v_{1,2}(\mathbf{r})$:

$$g_{\mu\nu}(\mathbf{r}) = \begin{bmatrix} -v_1(\mathbf{r})\, v_2(\mathbf{r}) & 0 & 0 \\ 0 & \frac{v_2(\mathbf{r})}{v_1(\mathbf{r})} & 0 \\ 0 & 0 & \frac{v_1(\mathbf{r})}{v_2(\mathbf{r})} \end{bmatrix}$$

Scalar curvature for $v_1(\mathbf{r}) = v_2(\mathbf{r}) = v(\mathbf{r})$:

$$R = -\frac{2}{v(\mathbf{r})} \nabla^2 v(\mathbf{r})$$

2+1-D Dirac fermion propagating through curved spacetime:

$$S = \int dt\, d^2\mathbf{r} \left[\bar{\psi}\, i\partial_t\, \psi + \frac{1}{2} \sum_{a=1,2} v_a(\mathbf{r}) \left(\bar{\psi}\, i\hat{\sigma}^a\, \overleftrightarrow{\partial}_a\, \psi \right) \right]$$

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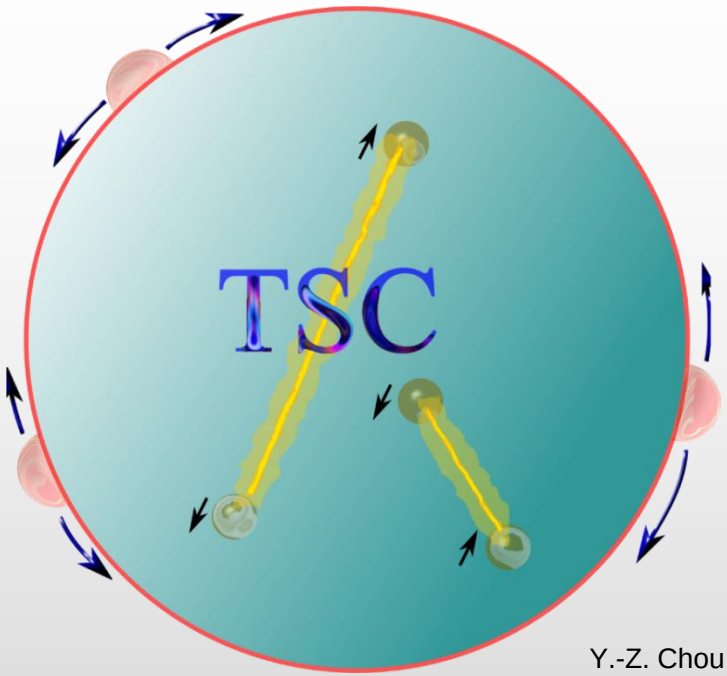
3D Topological Superconductivity

How to realize?

(1) Surface of class DIII topological superconductor



Y.-Z. Chou
Ph.D Thesis
(2016)

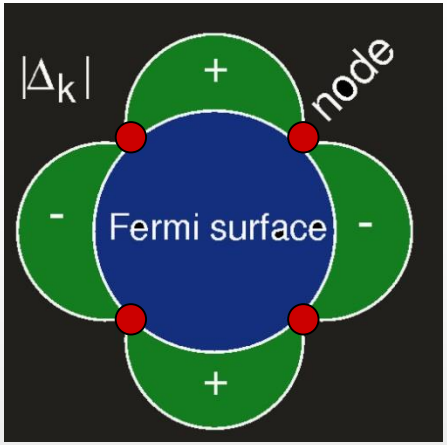
Electric potentials on the surface couple gravitationally:

$$v_1(\mathbf{r}) = v_2(\mathbf{r}) = v_0 \left[1 + \vartheta \frac{eA^0(\mathbf{r})}{E_{\text{bulk}}} \right]$$

Nakai and Nomura (2014)
Ghorashi and Foster (2019)

2D Quasiparticles in a correlated phase

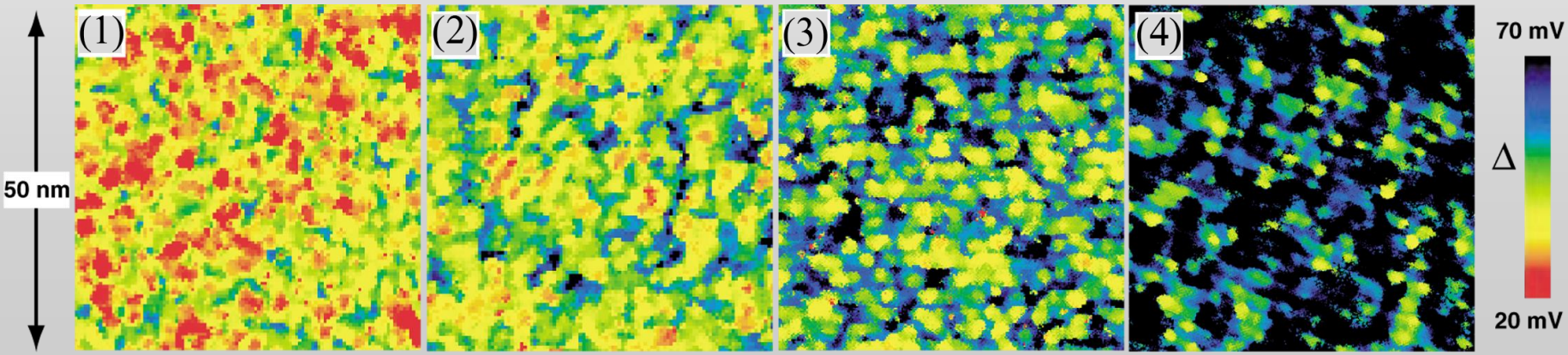
How to realize?



P. Hirschfeld

(2) Spatial order parameter fluctuations in a correlated phase

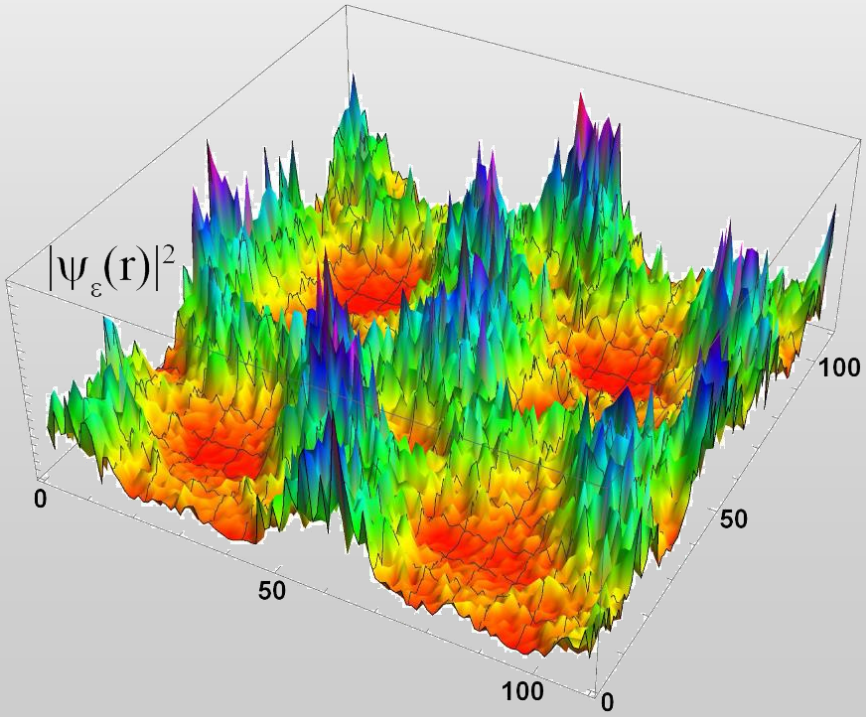
D-wave cuprates: $\Delta(r)$ modulates quasiparticle velocities parallel to the Fermi surface



BSCCO STM Data: K. McElroy, J. C. Davis *et al.*, PRL (2005)

2D Dirac material with velocity modulation

Low energy:
<u>Disorder irrelevant</u>.
Plane-wave-like
(ergodic) states



Finite energy:
<u>Disorder irrelevant</u>.
Energy relevant.
... *Ergodic states ?*

Nakai and Nomura, 2014

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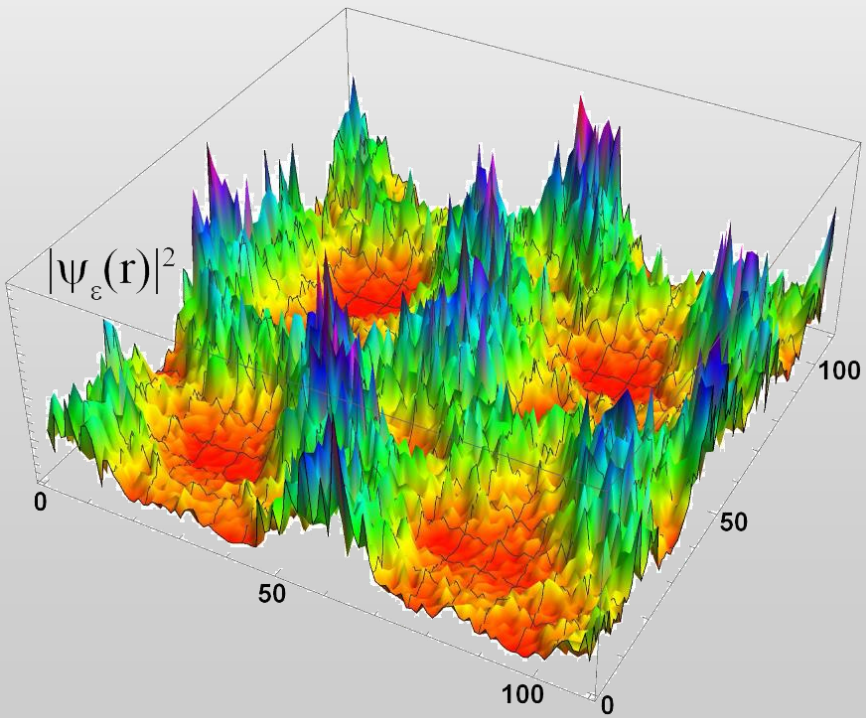
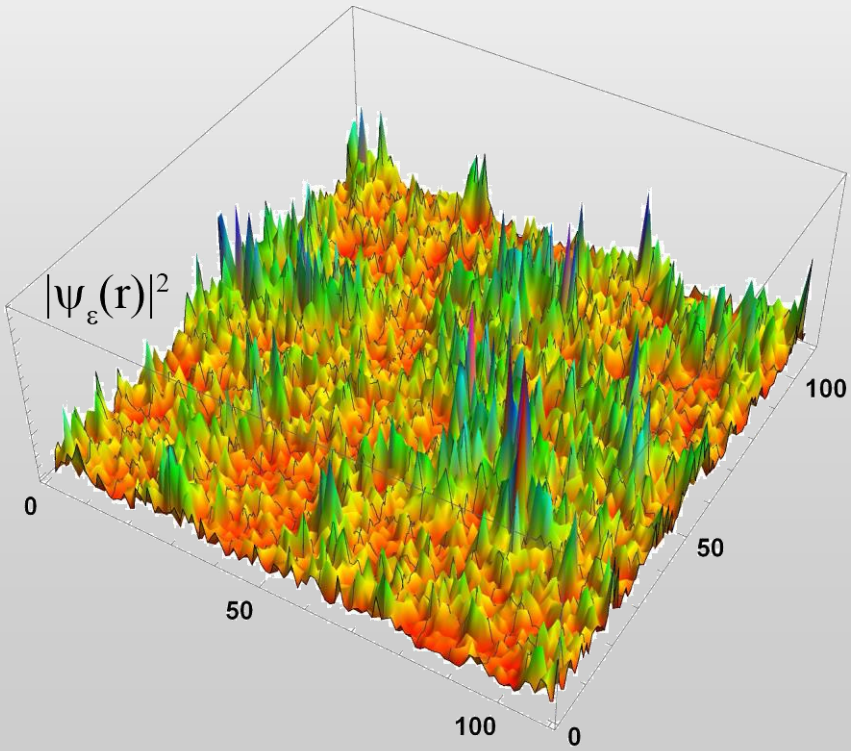
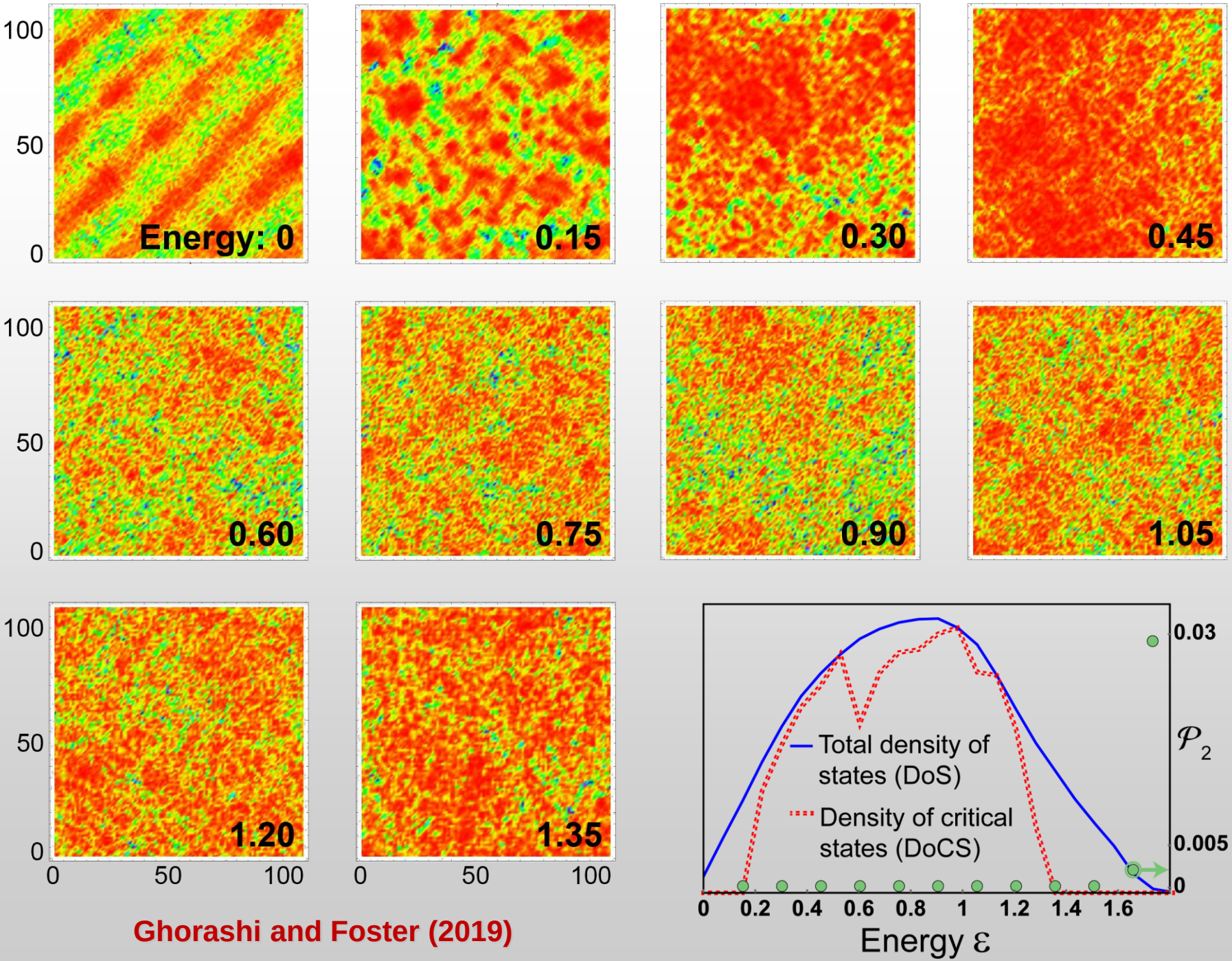


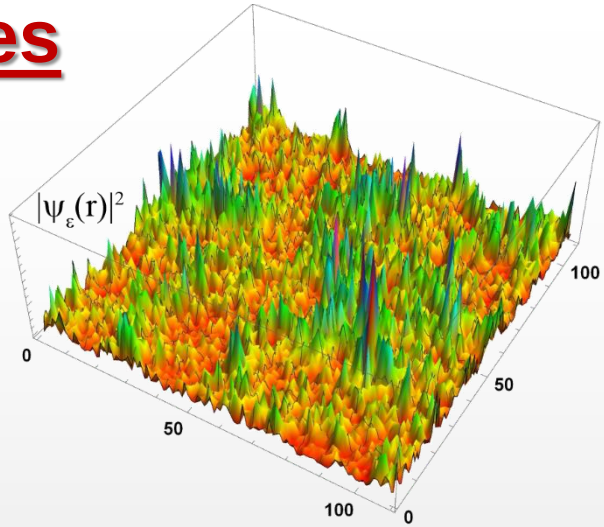
Finite energy:
<u>Disorder irrelevant</u>.
Energy relevant.
(Lots of) Critical states!





Ghorashi and Foster (2019)

Multifractal analysis of critical states



$$\int d^2\mathbf{r}\,|\psi(\mathbf{r})|^{2q} \sim \left(\frac{b}{L}\right)^{\tau(q)}$$

- Moment spectrum:

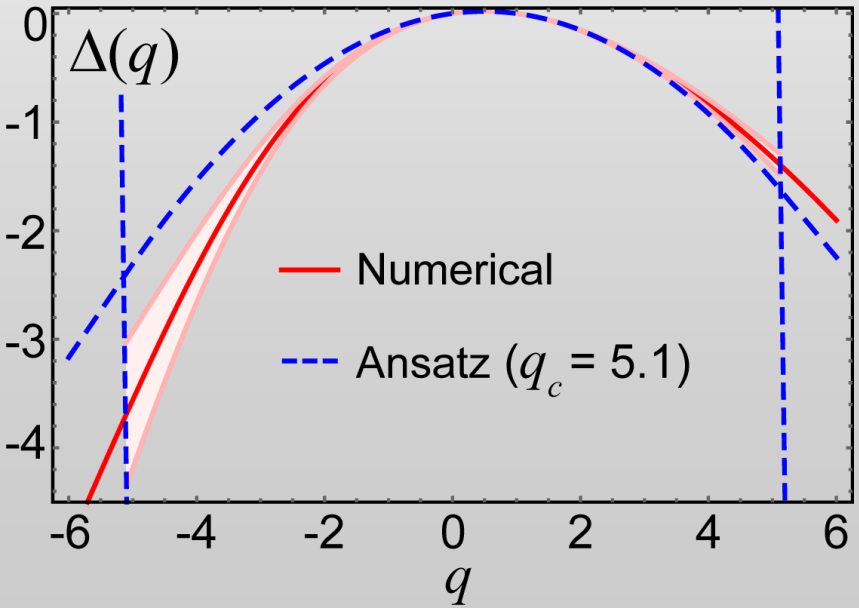
$$\tau(q) = \begin{cases} 2(q-1), & \text{plane wave} \\ 0, & \text{localized state} \end{cases}$$

- Anomalous part:

$$\Delta(q) \equiv \tau(q) - 2(q-1)$$

- Parabolic ansatz:

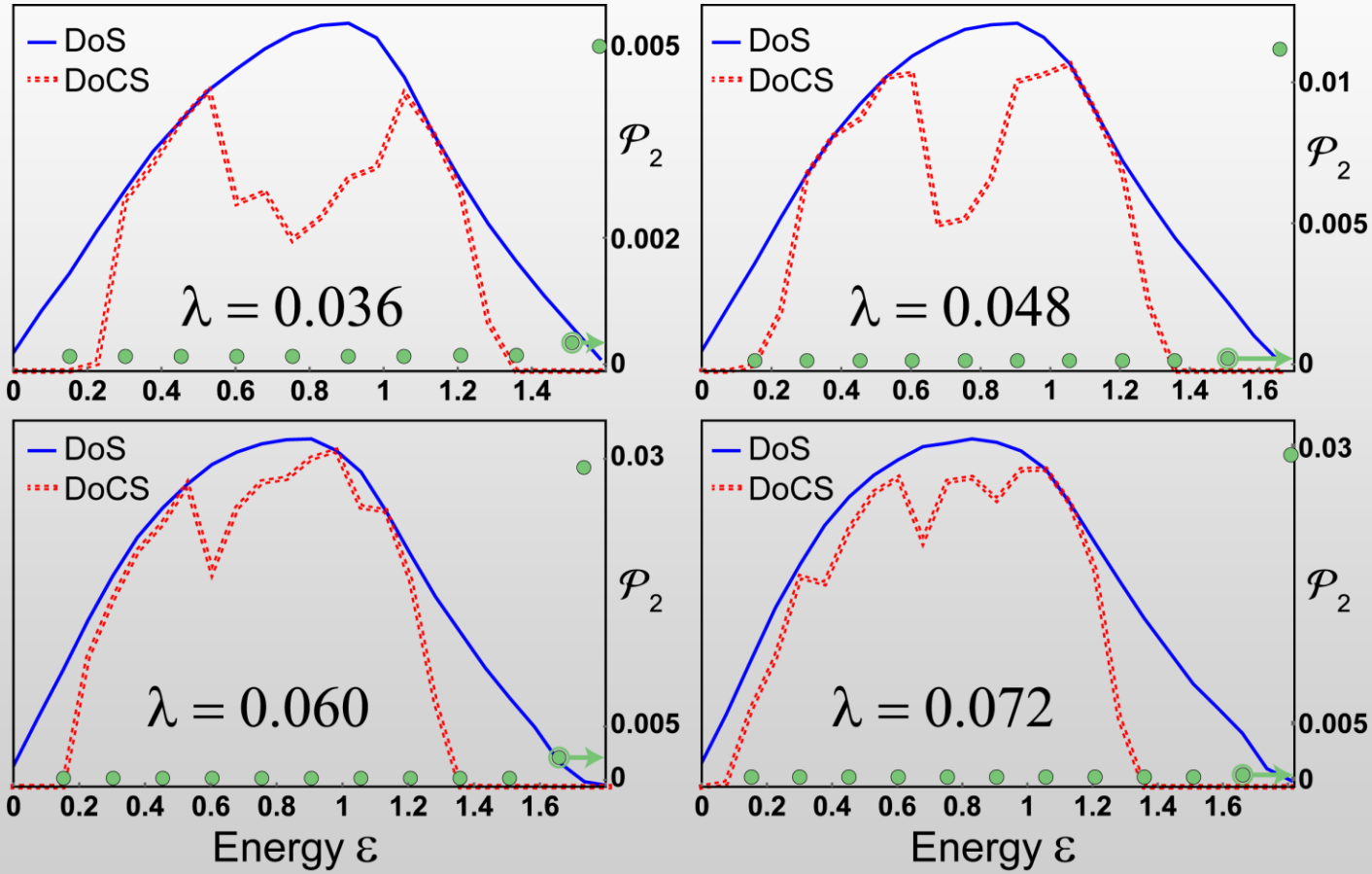
$$\Delta(q) = \frac{2}{q_c^2} q(1-q)$$



Disorder dependence:

Ghorashi and Foster (2019)

Critical states with universal statistics across the energy spectrum (State counted in DoCS: 4% error threshold, 85% $0 < q < q_c$)



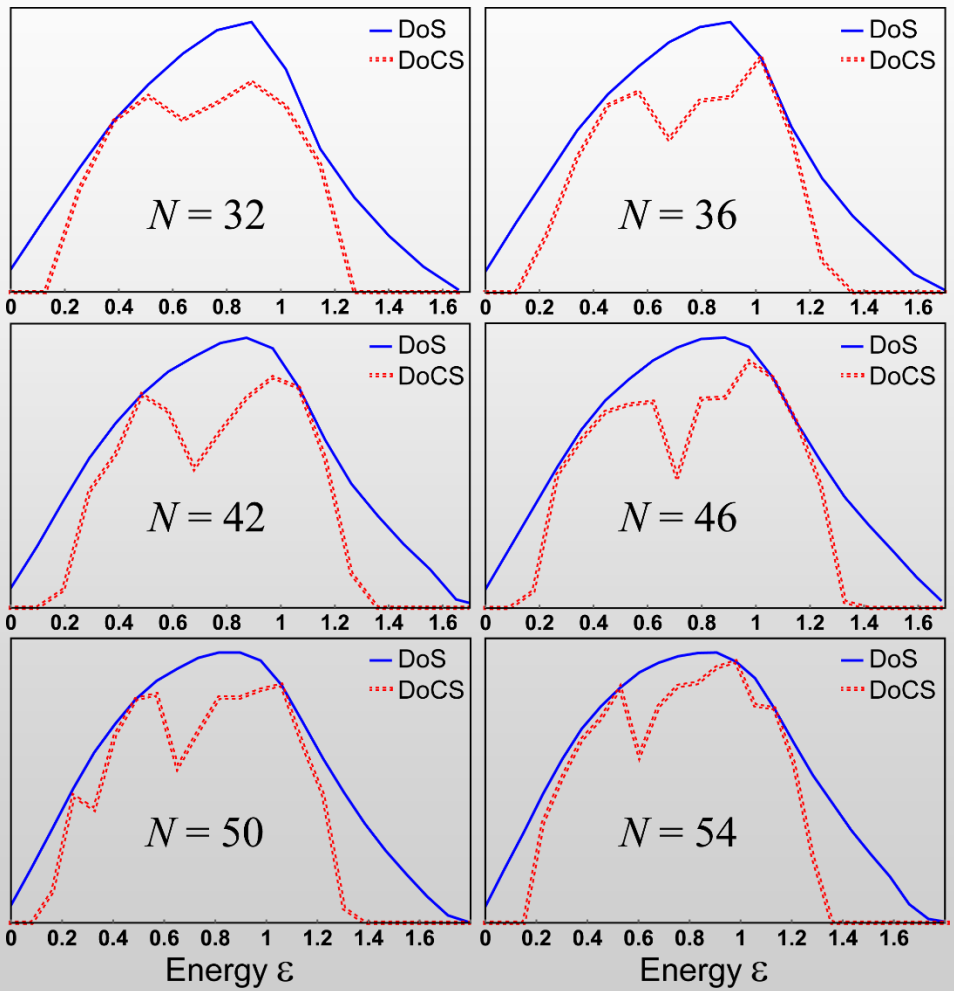
Strong disorder (horizons): $\Delta v_{1,2} \gtrsim v_0 \quad \Rightarrow \quad \lambda \gtrsim 0.4$

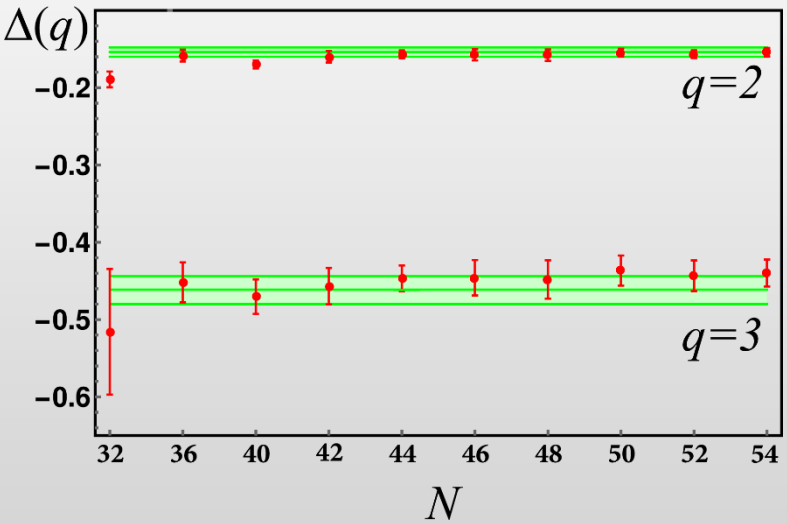
System-size dependence:

Ghorashi and Foster (2019)

Size = (2N+1)x(2N+1) momentum grid

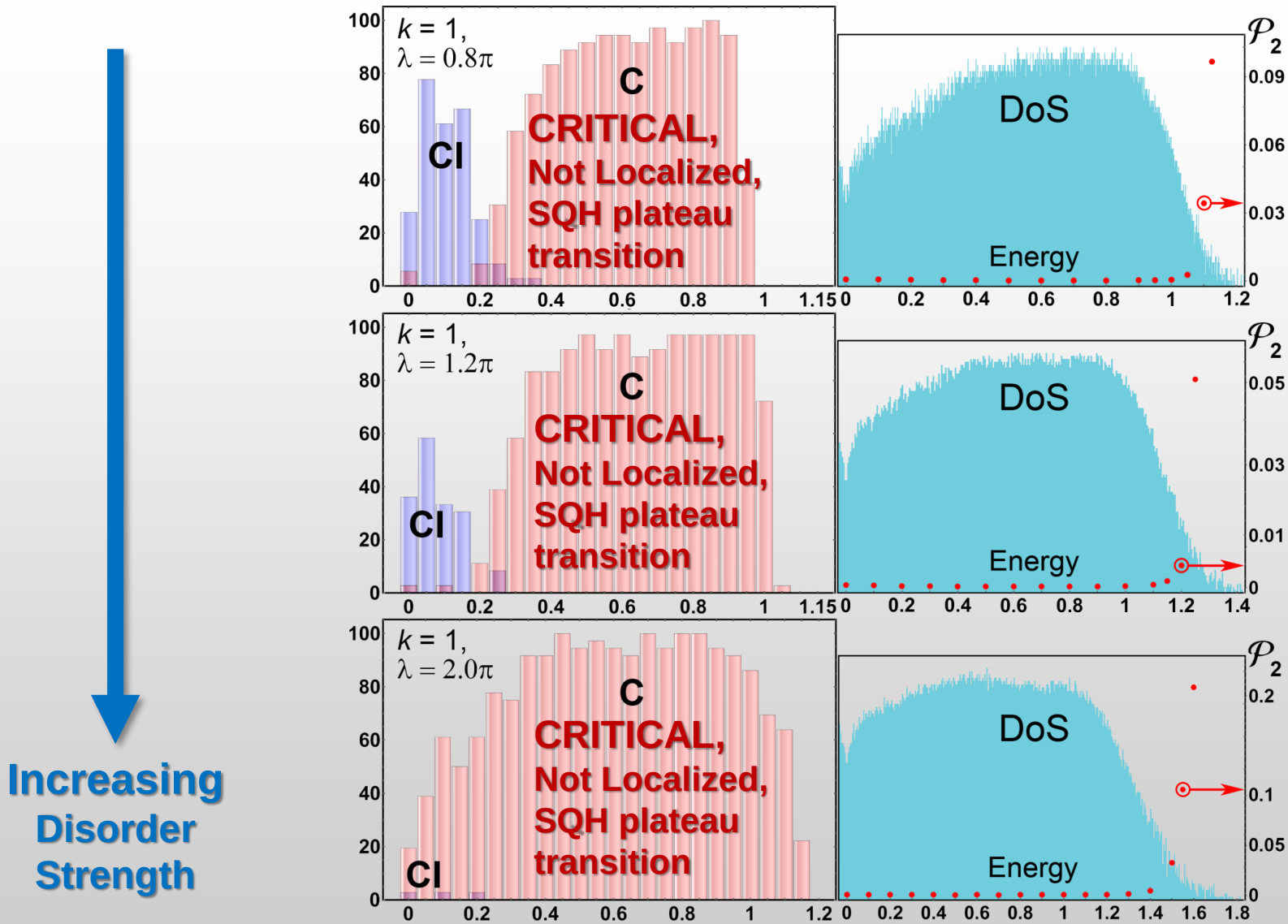
(State counted in DoCS: 4% error threshold, 85% 0 < q < q_c)





$$\lambda = 0.060$$

Compare: Dirac TSC surface with random SU(2) gauge dirt (CI)



Critical Percolation without Fine-Tuning on the Surface of a Topological Superconductor

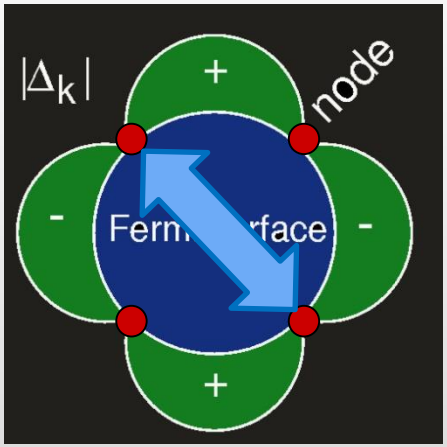
Ghorashi, Liao, and Foster PRL **121**, 016802 (2018)

Compare: Dirac TSC surface with random SU(2) gauge dirt (CI)

Topological CI (WZNW):

Equivalent to scattering between *pairs* of nodes in cuprates:

Nersesyan, Tsvelik, Wenger PRL (1994)
Schnyder, Ryu, Ludwig PRL (2009)



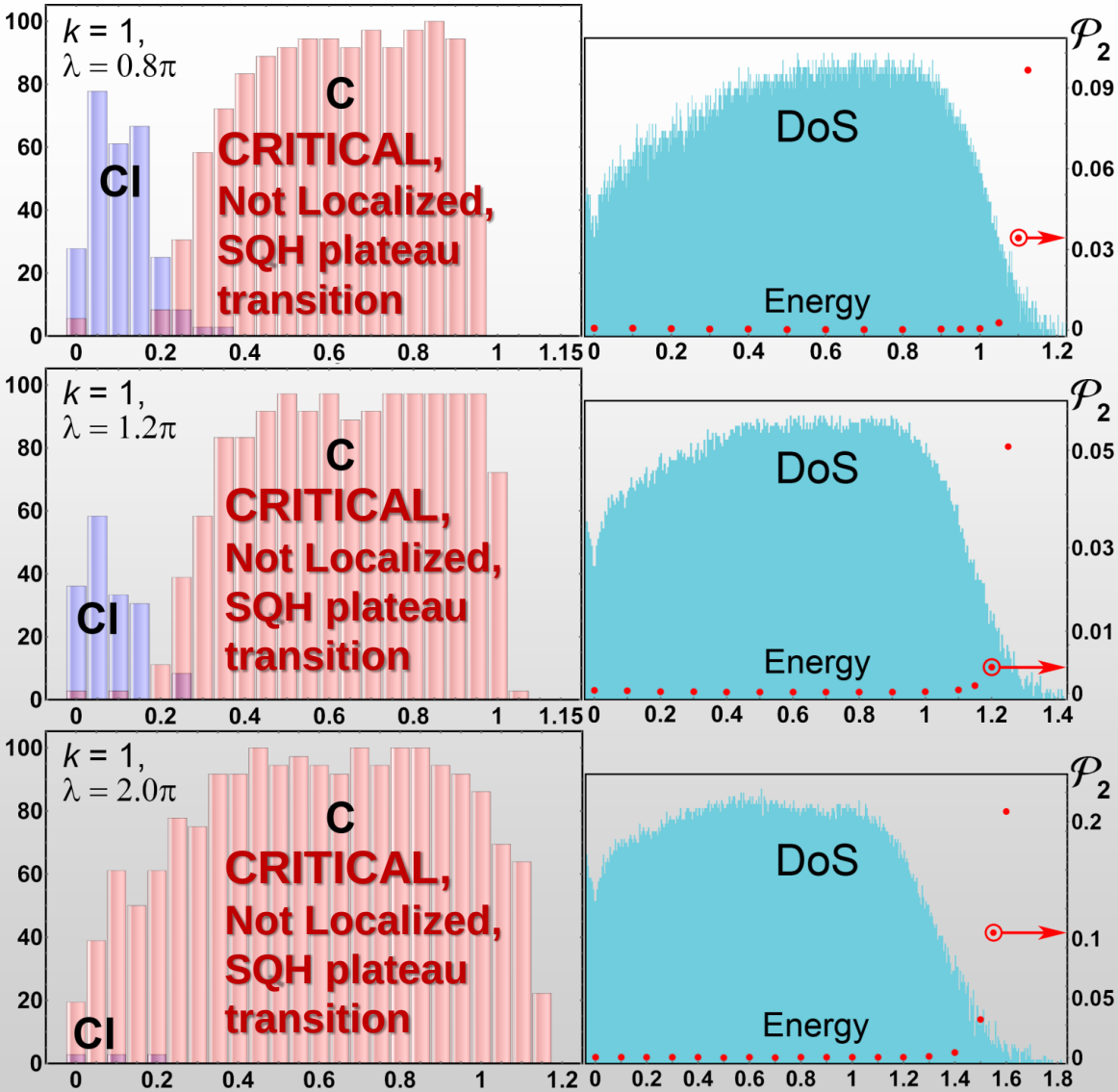
Strongest multifractality predicted at <u>zero energy</u>

Generic CI (no WZNW):

Scatter between all four nodes.

All states localized

Senthil, Fisher, Balents, Nayak PRL (1998)
Altland, Simons, Zirnbauer (2002)



Ghorashi, Liao, and Foster, PRL **121**, 016802 (2018)

Summary: Bulk TSCs and 2D Majorana surface fluid theory

1. **Odd-parity, topological pairing natural in Dirac semimetals (Fu, Berg 2010), d-wave, Weyl pairing natural in Luttinger semimetals, but odd-parity TSC is possible [Savary, Ruhman, Venderbos, Fu, Lee (2017), 1]**

2. **Power-law temperature-suppression of Meissner penetration depth due to paramagnetic response of gapless Majorana surface fluid [2]**

3. **Quantized heat and (if conserved) spin transport (no Altshuler-Aronov corrections) [3]**

4. **Criticality [4] throughout the energy spectrum, *without fine-tuning*, on the surface of a dirty TSC: Robust multifractality, artificial gravity [5] QCD [6].**

- **Criticality: "stacks" of quantum Hall plateau transition states in C, A, D [5,6]?**
- **Random lensing results similar to cuprate STM data [5]**

1. Roy, Ghorashi, Foster, Nevidomskyy PRB (2019)
2. Wu, Pal, Hosur, Foster, **to appear**
3. Xie, Chou, Foster PRB (2015)
4. Foster and Yuzbashyan PRL (2012); Foster, Xie, Chou PRB (2014); Chou and Foster PRB (2014)
5. Ghorashi and Foster arXiv (2019), under review @ PRL
6. Ghorashi, Liao, and Foster PRL (2018)