Quenched BCS superfluids:
Topology and spectral probes

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Quenched BCS superfluids: Topology and spectral probes

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[1] Rice University [2] Kent State University [3] University of Colorado, Boulder [4] Rutgers University

1. Foster, Dzero, Gurarie, and Yuzbashyan — PRB 2013
2. Foster, Gurarie, Dzero, and Yuzbashyan — PRL 2014
3. Liao and Foster — PRA 2015
4. Chou, Liao, and Foster — (unpublished)

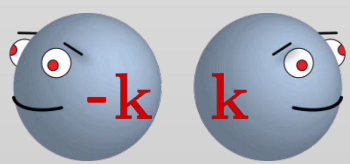
Spin-polarized fermions in 2D: P-wave BCS Hamiltonian

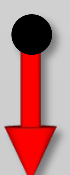
$$H = \sum_{\mathbf{k}} \frac{k^2}{2m} c^\dagger_{\mathbf{k}} c_{\mathbf{k}} - \frac{G}{2m} \sum_{\mathbf{k},\mathbf{k}'} \mathbf{k}\cdot\mathbf{k}'\, c^\dagger_{\mathbf{k}} c^\dagger_{-\mathbf{k}}\, c_{-\mathbf{k}'} c_{\mathbf{k}'}$$

Anderson pseudospins

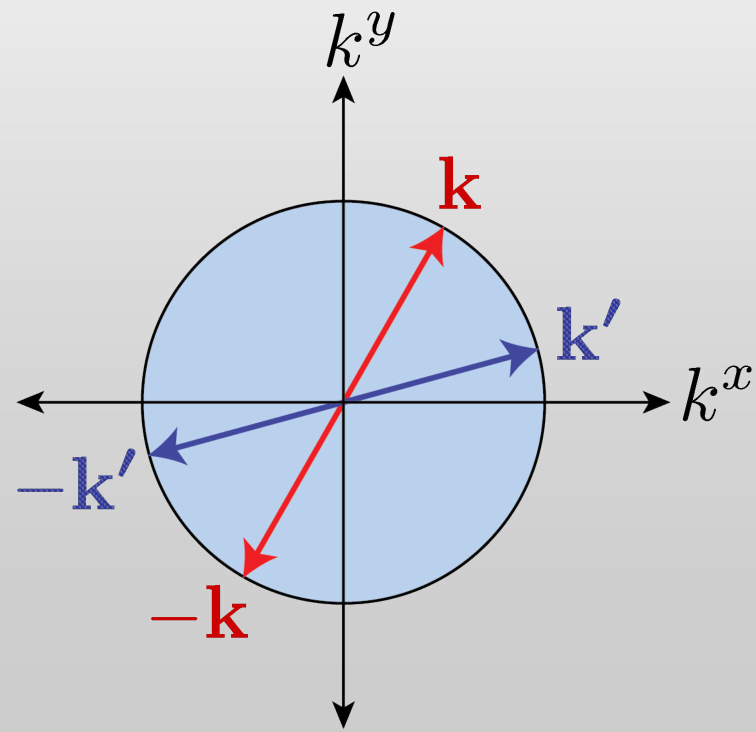
$$s^z_{\mathbf{k}} \equiv \frac{1}{2}\left[c^\dagger_{\mathbf{k}} c_{\mathbf{k}} + c^\dagger_{-\mathbf{k}} c_{-\mathbf{k}} - 1\right]$$

$$s^-_{\mathbf{k}} \equiv c_{-\mathbf{k}} c_{\mathbf{k}}, \quad s^+_{\mathbf{k}} \equiv c^\dagger_{\mathbf{k}} c^\dagger_{-\mathbf{k}}$$

$\langle s^z_{\mathbf{k}} \rangle = +\frac{1}{2}$ $\Big|$  $\Big\rangle$



$\langle s^z_{\mathbf{k}} \rangle = -\frac{1}{2}$ $\Big|$ {k,-k} vacant $\Big\rangle$



P + i p topological superconductivity in 2D

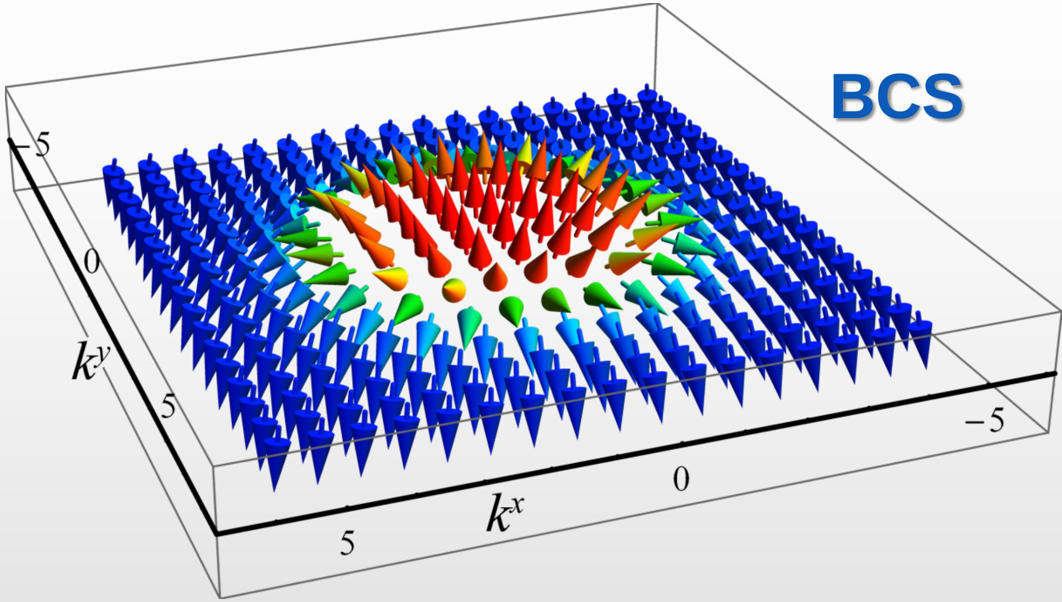
Pseudospin winding number *Q* :

$$\rho(\mathbf{k}) = \frac{2\epsilon_{abc}}{\pi k}\langle s^a_{\mathbf{k}}\rangle \partial_k \langle s^b_{\mathbf{k}}\rangle \partial_{\phi_k}\langle s^c_{\mathbf{k}}\rangle$$

$$Q \equiv \int_{\mathbf{k}} \rho(\mathbf{k}) = [\langle s^z_{\mathbf{k}=0}\rangle - \langle s^z_{\mathbf{k}=\infty}\rangle]$$

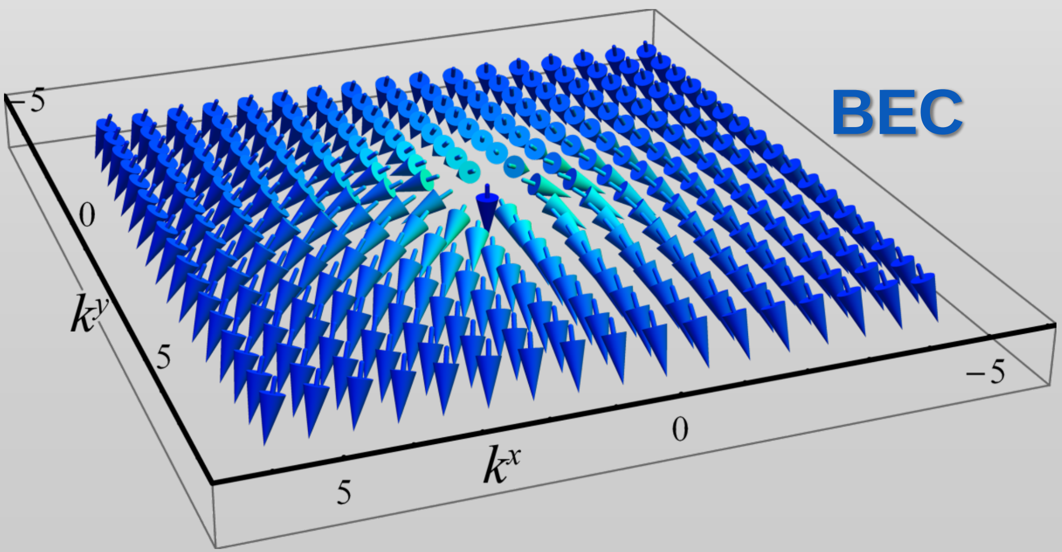
$$= \begin{cases} 1, & \mu > 0 \text{ (BCS)} \\ 0, & \mu < 0 \text{ (BEC)} \end{cases}$$

Volovik 88; Read and Green 00



2D Topological superconductor

- **Fully gapped when $\mu \neq 0$**
- ***Weak-pairing* BCS state topologically non-trivial**
- ***Strong-pairing* BEC state topologically trivial**



P + i p topological superconductivity in 2D

Pseudospin winding number *Q* : “Topology of the state”

$$\rho(\mathbf{k}) = \frac{2\epsilon_{abc}}{\pi k}\langle s^a_{\mathbf{k}}\rangle \partial_k \langle s^b_{\mathbf{k}}\rangle \partial_{\phi_k}\langle s^c_{\mathbf{k}}\rangle$$

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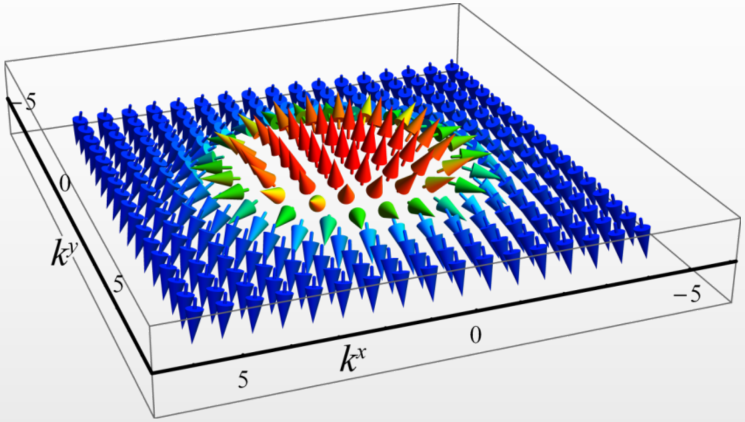
$$= \begin{cases} 1, & \mu > 0 \text{ (BCS)} \\ 0, & \mu < 0 \text{ (BEC)} \end{cases}$$



Volovik 88; Read and Green 00

Retarded GF winding number *W* : “Topology of the effective single particle Hamiltonian”

$$W \equiv \frac{\pi\epsilon^{\alpha\beta\gamma}}{3}\mathrm{Tr}\int_{\omega,\mathbf{k}} \left(\hat{G}^{-1}\partial_{k^\alpha}\hat{G}\right)\left(\hat{G}^{-1}\partial_{k^\beta}\hat{G}\right)\left(\hat{G}^{-1}\partial_{k^\gamma}\hat{G}\right)$$

Niu, Thouless, and Wu 85
Volovik 88

$$i\frac{\partial G}{\partial t} - HG = \delta(t-t'), \quad H = \begin{bmatrix} \frac{p^2}{2m} - \mu & \Delta\, p\, e^{i\phi_{\mathbf{p}}} \\ \Delta^*\, p\, e^{-i\phi_{\mathbf{p}}} & -\frac{p^2}{2m} + \mu \end{bmatrix}$$

- ***W* = *Q* in equilibrium**

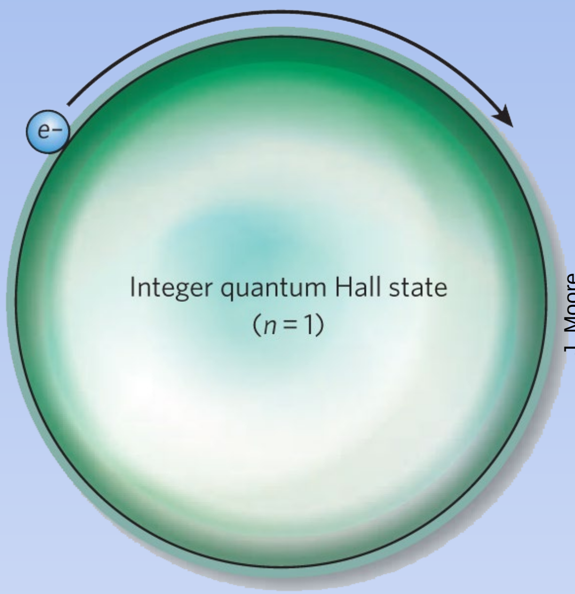
P + i p topological superconductivity in 2D

Topological signatures: *Majorana fermions*

1. **Chiral 1D Majorana edge states quantized thermal Hall conductance**

$$\kappa_{xy} = c\frac{\pi^2 k_B T}{6\pi\hbar}$$

2. **Isolated Majorana zero modes (vortices)**



Realizations?

- **Cold atoms: ^{6}Li, ^{40}K**

Gurarie, Radzihovsky, Andreev 05; Gurarie and Radzihovsky 07

PROBLEM: Losses due to three-body processes

$$t_3 \sim \frac{l}{b} t_F \sim 200 t_F \sim 20 \text{ ms in } {}^6\text{Li}$$

$l \sim 1000$ nm interparticle sep

$b \sim 5$ nm Van der Waals length

Decay is too fast for adiabatic cooling

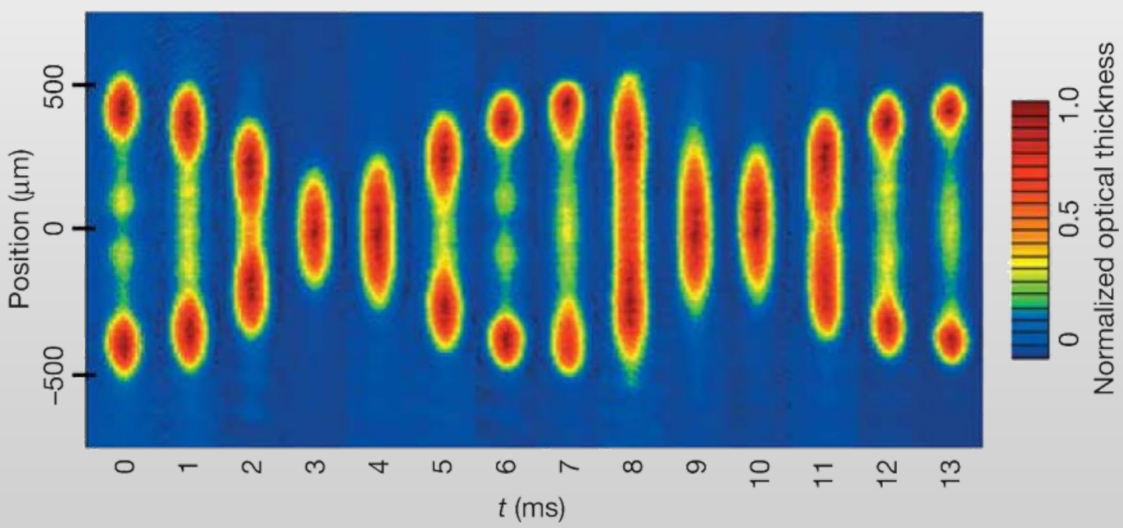
Zhang et al. 04 Jona-Lasinio, Pricoupenko, Castin 08
Levinsen, Cooper, Gurarie 08 Gaebler, Stewart, Bohn, Jin 07
Fuchs et al. 08, Inada et al. 08

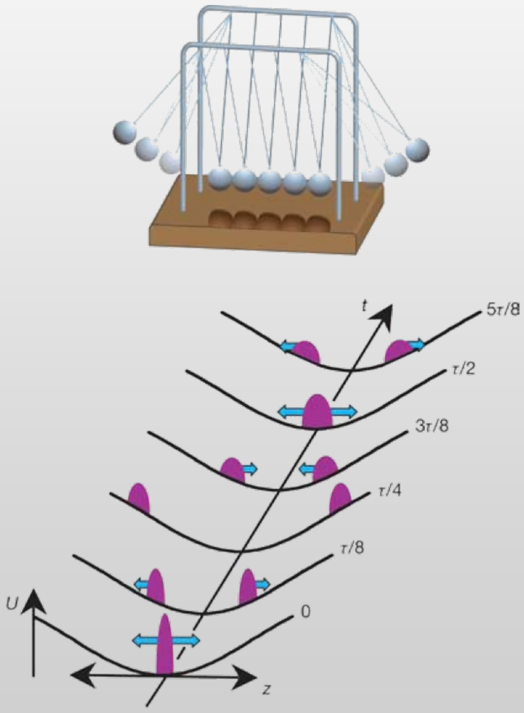
➢ **SOLUTION: Forget equilibrium…?** $t_F \lesssim t \lesssim t_3$

Experimental Example:

Quantum Newton's Cradle for trapped 1D ^{87}Rb Bose Gas

Kinoshita, Wenger, and Weiss 06





Exact quench phase diagram: Strong to weak, weak to strong quenches

Gap dynamics similar to s-wave case

Barankov, Levitov, Spivak 04, Warner and Leggett 05
Yuzbashyan, Altshuler, Kuznetsov, Enolskii 05, Yuzbashyan, Tsyplyatyev, Altshuler 05
Barankov and Levitov, Dzero and Yuzbashyan 06

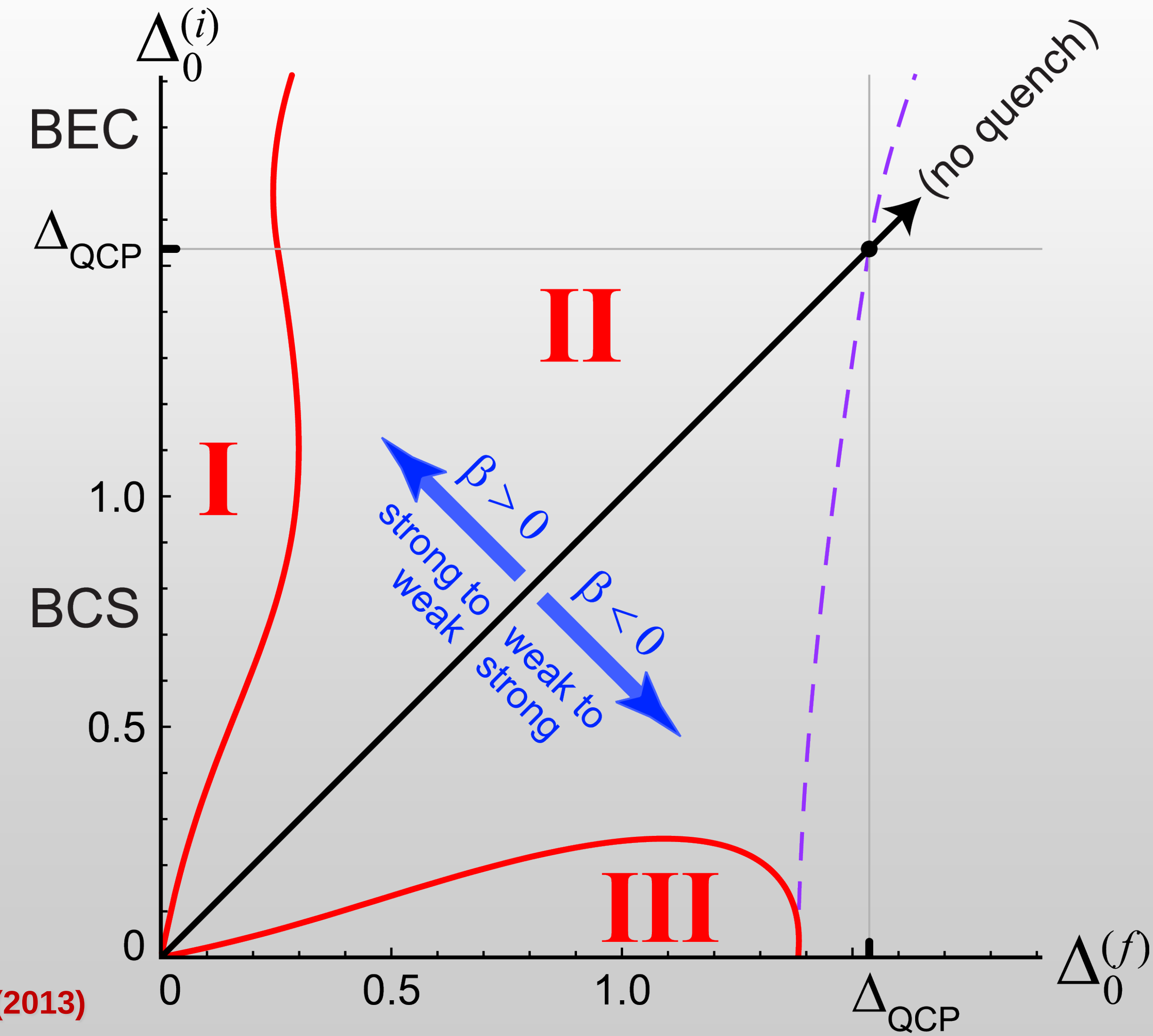
Dynamical phases: $\Delta(t \to \infty)$

Phase I:
Gap decays to zero.

Phase II:
Gap goes to a constant.

Phase III:
Gap oscillates.

Foster, Dzero, Gurarie, Yuzbashyan (2013)



Phase III quench dynamics: Oscillating gap

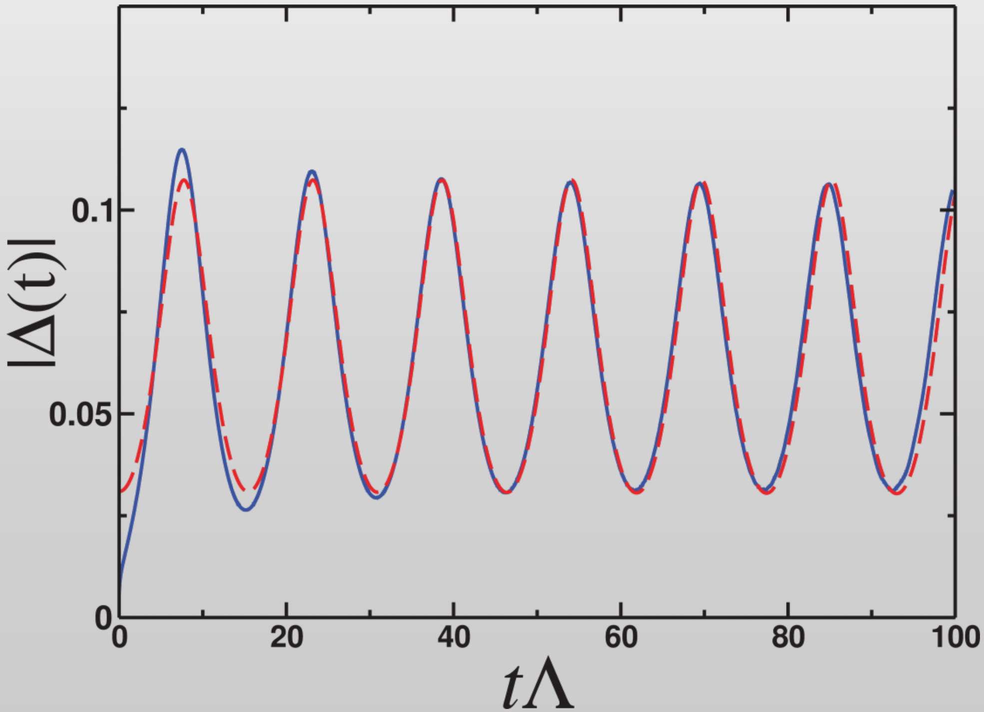
Initial parameters:

$$\mu_0^{(i)} = 0.99992\,\varepsilon_F$$
$$\Delta_0^{(i)} = 5.03 \times 10^{-3}$$
$$\varepsilon_F = 2\pi n = 5.18$$
$$\Delta_{\text{QCP}} = 1.54$$

Blue curve:
classical spin dynamics
(numerics 5024 spins)

Red curve:
Exact analytical solution

$$\Delta_0^{(f)} = 0.108,\ \mu_0^{(f)} = 0.990\,\varepsilon_F$$



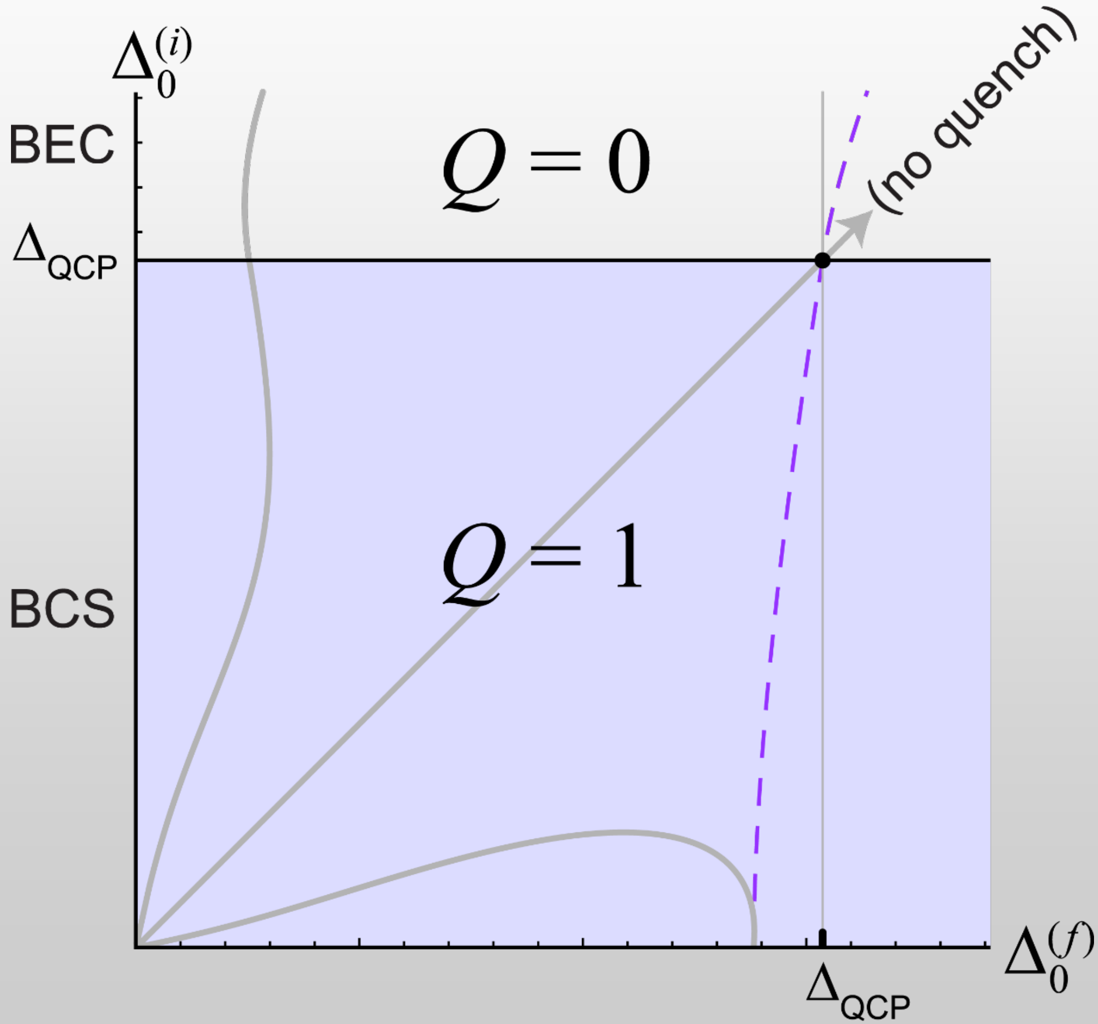
- **Quench-induced Floquet phase!**
- **No continuous driving!**

Pseudospin winding number *Q*: Unchanged by quench!

Smooth evolution of pseudospin texture:

Topology of the state *Q* doesn't change.

Not what we are usually interested in!



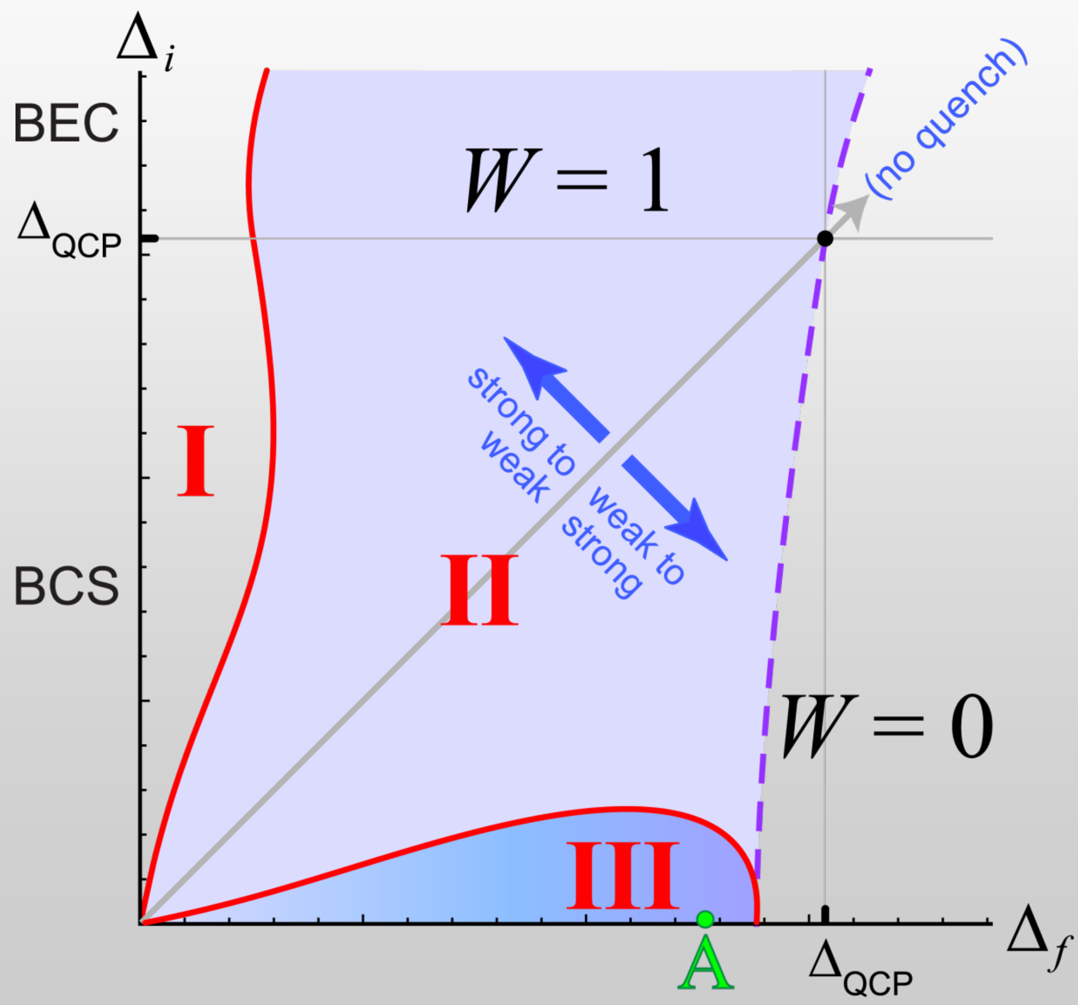
Foster, Dzero, Gurarie, Yuzbashyan (2013)
Y. Dong, L. Dong, M. Gong, and H. Pu (2014)
L. D'Alessio and M. Rigol (2015)

Retarded GF winding number *W*: Can change

Retarded GF winding number *W* :

$$i\frac{\partial G}{\partial t} - HG = \delta(t-t'),\ H = \begin{bmatrix} \frac{p^2}{2m} - \mu_\infty & \Delta_\infty\, p\, e^{i\phi_{\mathbf{p}}} \\ \Delta_\infty^*\, p\, e^{-i\phi_{\mathbf{p}}} & -\frac{p^2}{2m} + \mu_\infty \end{bmatrix}$$

- **Topology of the Bogoliubov-de Gennes Hamiltonian**
- **Signals presence of Majorana edge modes**



"**winding**":
Majorana edge modes

- $W = 1$

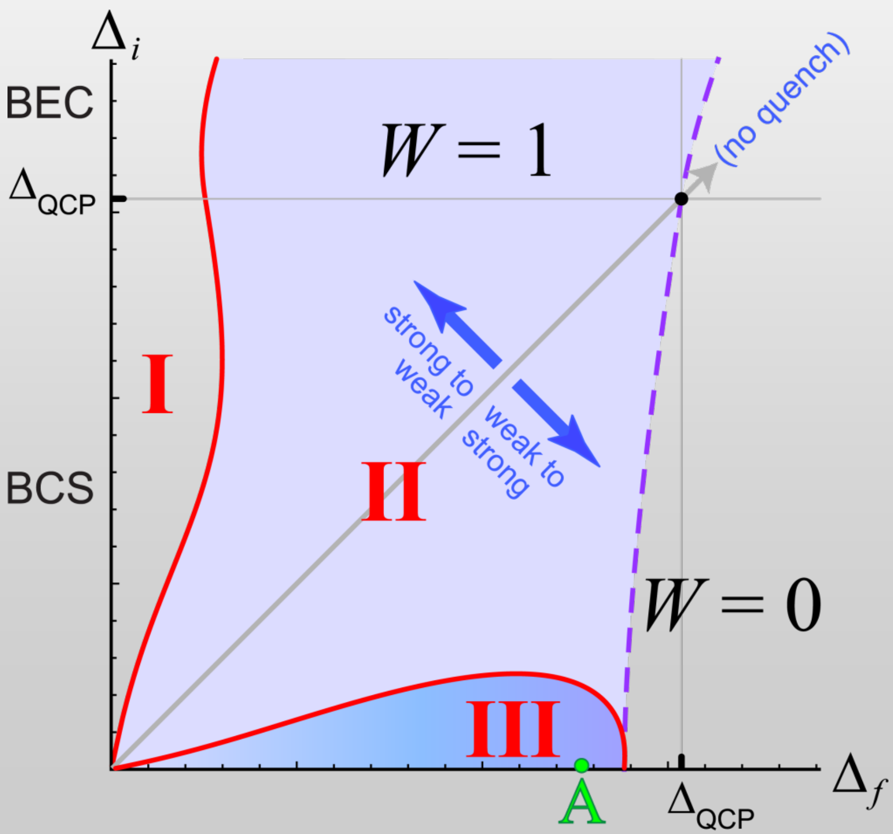
"**non-winding**"

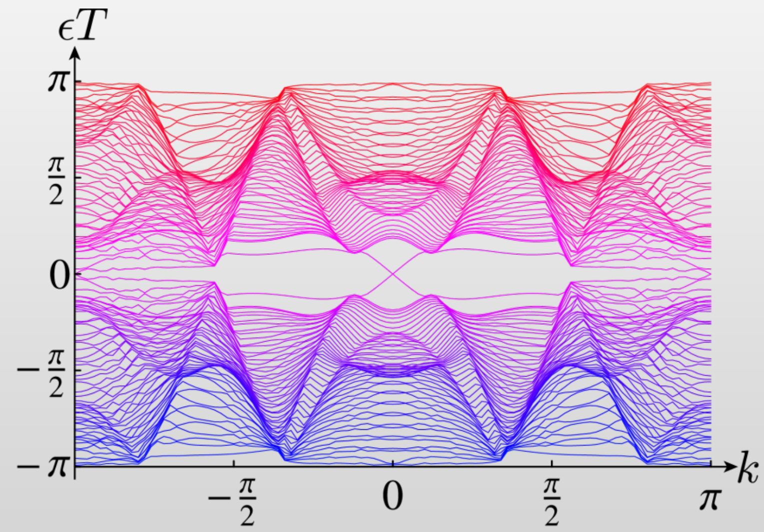
- $W = 0$

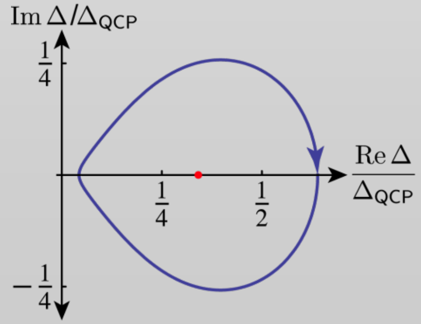
Phase III: Quench-induced Floquet topological superconductor

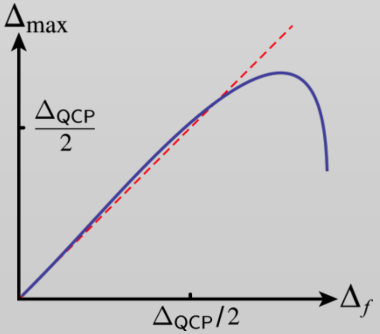
$$i\frac{\partial G}{\partial t} - HG = \delta(t-t'),\ \ H = \begin{bmatrix} \frac{p^2}{2m} - \mu_\infty & \Delta_\infty\, p\, e^{i\phi_{\mathbf{p}}} \\ \Delta_\infty^*\, p\, e^{-i\phi_{\mathbf{p}}} & -\frac{p^2}{2m} + \mu_\infty \end{bmatrix}$$

- **Weak-to-strong quench in III: $\Delta_\infty(t + T) = \Delta_\infty(t)$**
- **Asymptotic GF same as for an externally driven Floquet system**









Phase III: Quench-induced Floquet topological superconductor

$$i\frac{\partial G}{\partial t} - HG = \delta(t-t'), \;\; H = \begin{bmatrix} \frac{p^2}{2m} - \mu_\infty & \Delta_\infty \, p \, e^{i\phi_{\mathbf{p}}} \\ \Delta_\infty^* \, p \, e^{-i\phi_{\mathbf{p}}} & -\frac{p^2}{2m} + \mu_\infty \end{bmatrix}$$

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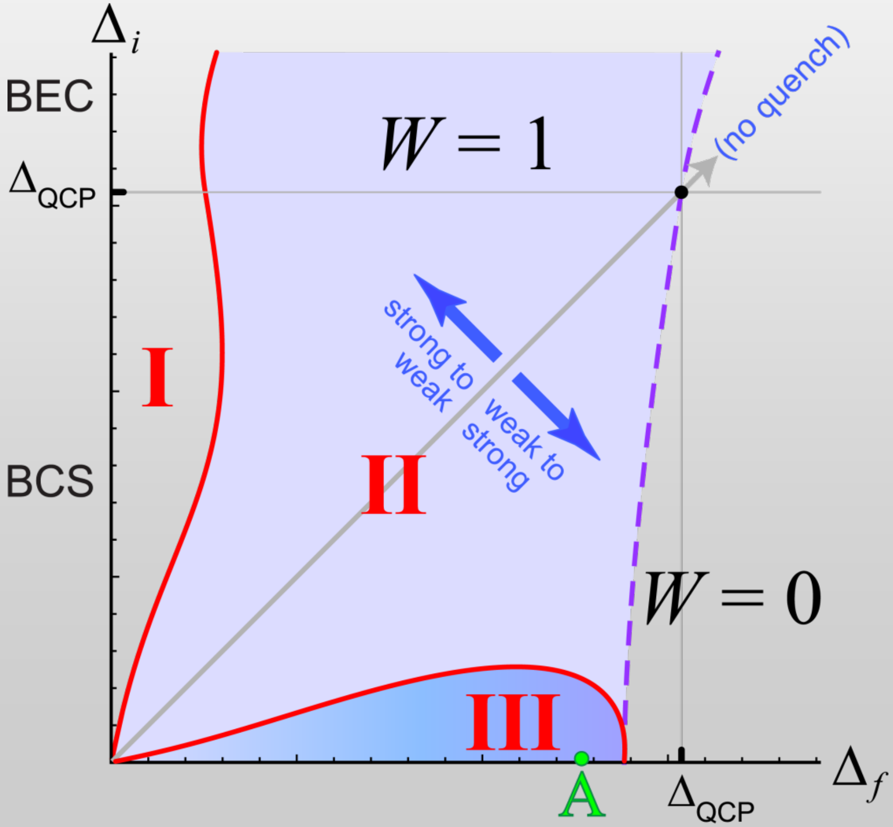


Phase III most relevant for cold atom experiments (?)

- **Prepare initial state with very weak p+ip order**
- **3-body losses negligible away from Feshbach resonance**

Jona-Lasinio, Pricoupenko, Castin 08

- **Quench to strong pairing**

$$t_F \lesssim t \lesssim \min(t_3, t_{\text{pb}})$$

Foster, Gurarie, Dzero, Yuzbashyan (2014)

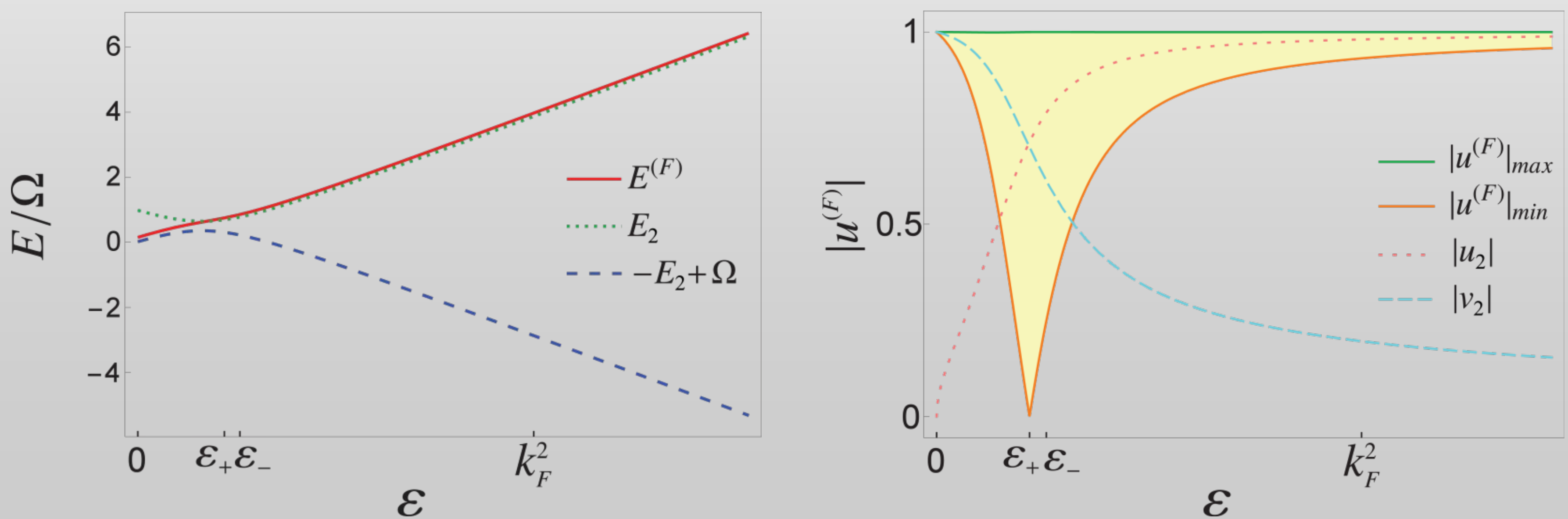
Spin-orbit + B: Y. Dong, L. Dong, M. Gong, and H. Pu (2014)

Asymptotic dynamics:

Liao and Foster (2015)

- **Superposition of "positive" and "negative" quasienergy Floquet states at each momentum *k***

$$\begin{bmatrix} u_{\mathbf{k}}(t) \\ v_{\mathbf{k}}(t) \end{bmatrix} = \sqrt{1-n(\mathbf{k})} \begin{bmatrix} u_{\mathbf{k}}^{(F)}(t) \\ v_{\mathbf{k}}^{(F)}(t) \end{bmatrix} e^{+iE_{\mathbf{k}}^{(F)}t} + \sqrt{n(\mathbf{k})} \begin{bmatrix} v_{\mathbf{k}}^{*(F)}(t) \\ -u_{\mathbf{k}}^{*(F)}(t) \end{bmatrix} e^{-iE_{\mathbf{k}}^{(F)}t+i\Gamma_{\mathbf{k}}}$$



Floquet phase III: Exact solution

Asymptotic dynamics:

Liao and Foster (2015)

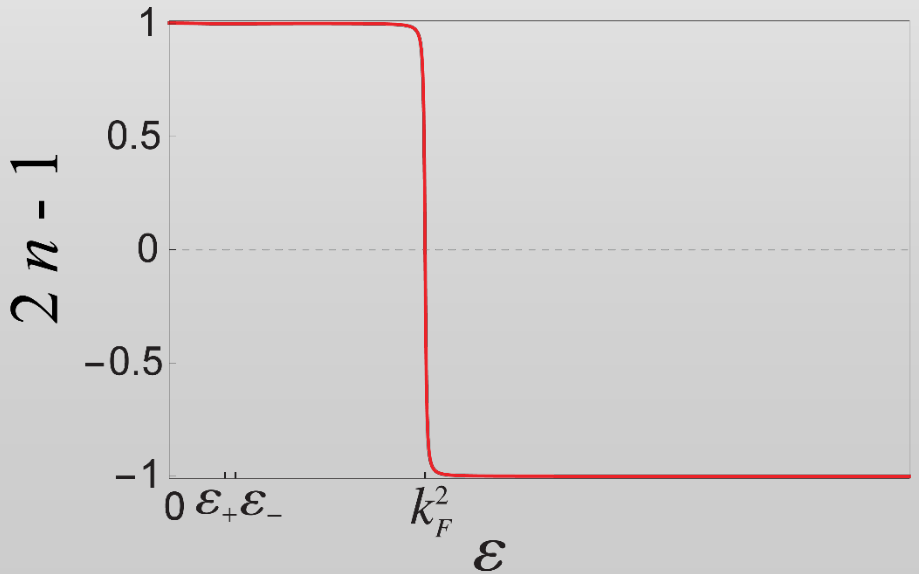
- **Superposition of "positive" and "negative" quasienergy Floquet states at each momentum *k***

$$\begin{bmatrix} u_{\mathbf{k}}(t) \\ v_{\mathbf{k}}(t) \end{bmatrix} = \sqrt{1 - n(\mathbf{k})} \begin{bmatrix} u_{\mathbf{k}}^{(F)}(t) \\ v_{\mathbf{k}}^{(F)}(t) \end{bmatrix} e^{+iE_{\mathbf{k}}^{(F)}t} + \sqrt{n(\mathbf{k})} \begin{bmatrix} v_{\mathbf{k}}^{*(F)}(t) \\ -u_{\mathbf{k}}^{*(F)}(t) \end{bmatrix} e^{-iE_{\mathbf{k}}^{(F)}t + i\Gamma_{\mathbf{k}}}$$

- ***Occupation factor n(k)* (distribution function)**

- **Population inversion of Floquet-Bloch states**

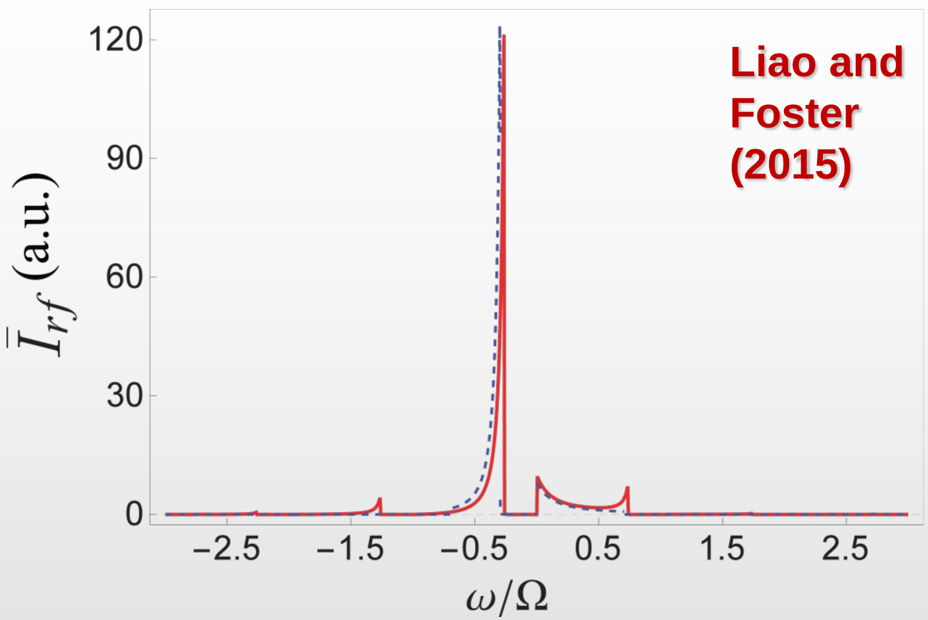
- **Related to BCS instability of normal state**



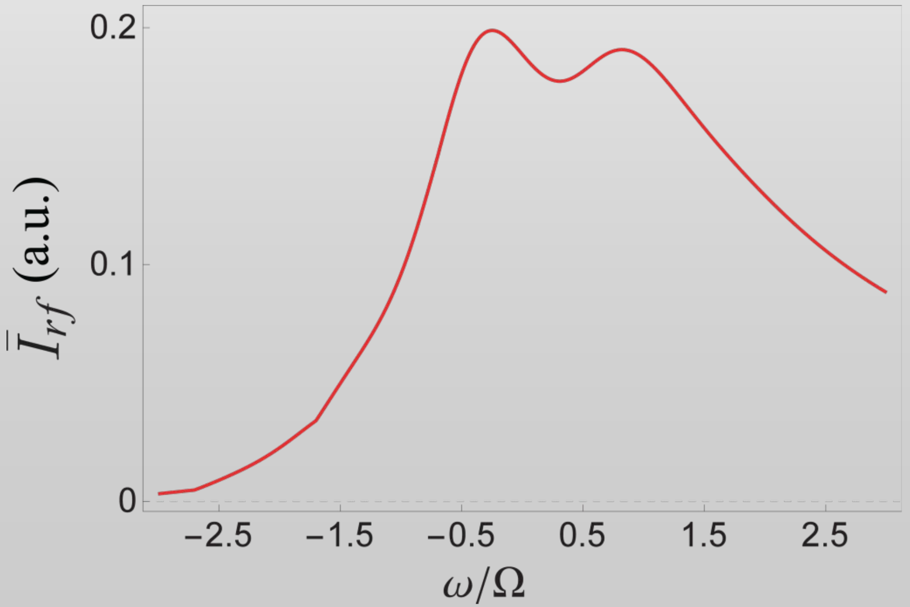
Floquet phase III: One-particle spectral measures

Distribution function: Crucial information

1. **Bulk RF spectrum, including physical *n*(*k*): Spectral Gap**



2. **Bulk RF spectrum, filling the "lower" Floquet band only *n*(*k*) = 0: no Gap**



Floquet phase III: One-particle spectral measures

Distribution function: Crucial information

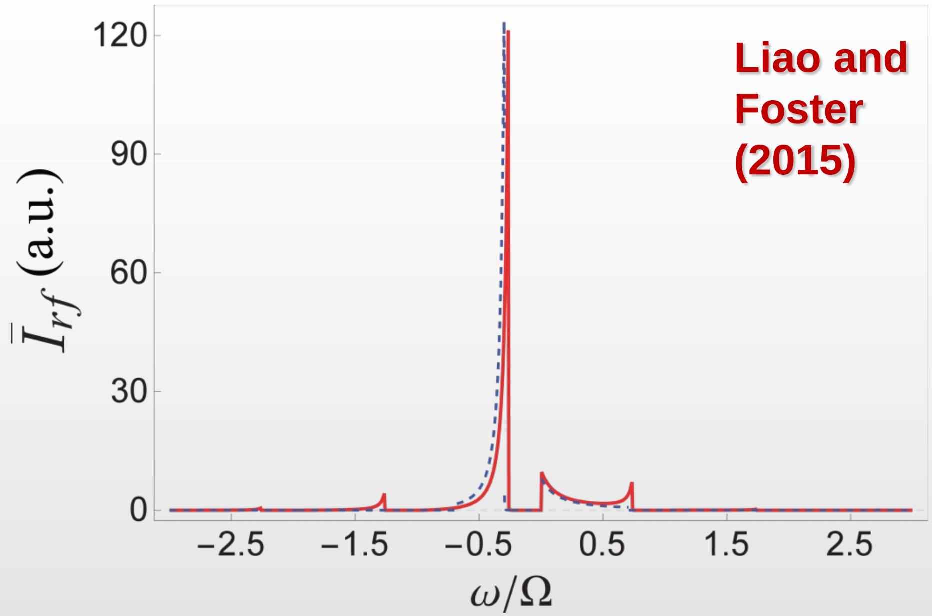
1. **Bulk RF spectrum, including physical *n*(*k*): Spectral Gap**

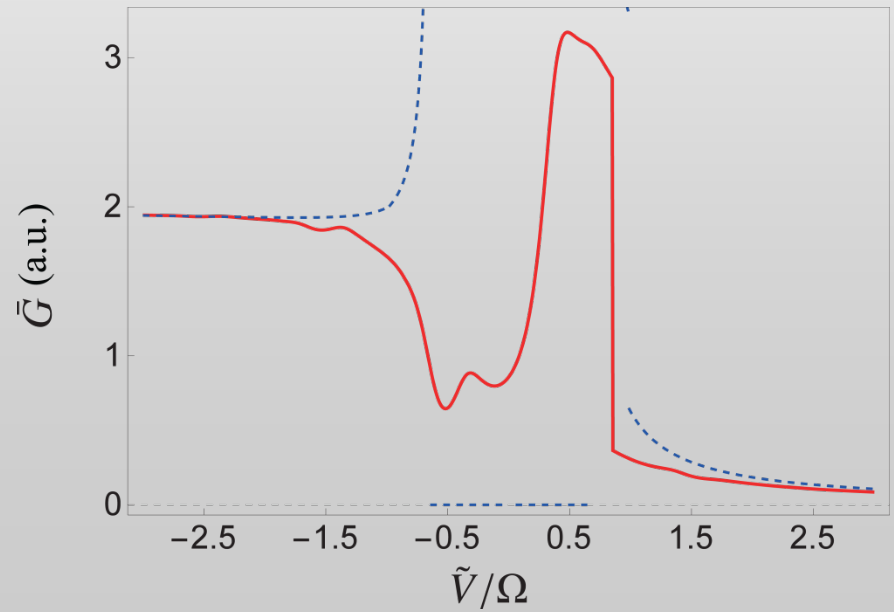
$$\sim G_{<}(\Omega, k)$$

2. **Tunneling spectrum, insensitive to distribution function: no Gap**

$$\sim G_{\mathrm{Ret}}(\Omega, k)$$

3. **ARPES—see paper**





Summary: 3 Dynamical phases of quenched BCS superfluids

- **Quench-induced Floquet phase III:**
  2D topological superfluid for p-wave pairing, ultracold atoms?

- **Different one-particle spectral measures**
  (RF vs. tunneling vs. ARPES) show different results, due to (in)sensitivity to nonequilibrium distribution function

1. Foster, Dzero, Gurarie, and Yuzbashyan — PRB 2013
2. Foster, Gurarie, Dzero, and Yuzbashyan — PRL 2014
3. Liao and Foster — PRA 2015
4. Chou, Liao, and Foster — (unpublished)