1. Dephasing catastrophe in $4 - \varepsilon$: A toy MBL-ergodic transition

2. Geometric percolation at the surface of a topological superconductor

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1. Liao, MSF arXiv: 1710.05037
• Can MBL occur in higher dimensions?
  MBL is stable in certain 1D systems: J. Z. Imbrie (2014)

• Nature of the MBL-ergodic transition in higher dimensions?
  1D MBL transition:
  - Vosk, Huse, Altman (2015)
  - Potter, Vasseur, Parameswaran (2015)
  - Serbyn and Moore (2015)

• Can MBL be destabilized by rare thermal fluctuations?
  de Roeck, Huveneers, Mueller, Schiulaz (2015)
• Weak localization correction (orthogonal metal class)

\[ \delta \sigma_{WL} = -\frac{8e^2}{h} D \int_{-\infty}^{\infty} d\eta \, \langle C_{\eta,-\eta}(x,x) \rangle_\rho = -\frac{e^2}{h \pi} \ln \left( \frac{\tau_\phi}{\tau_{el}} \right) \]

• Even a good metal \((E_F \tau_{el} \gg 1)\) would localize at all temperatures in 2D without dephasing \((\tau_\phi(T) < \infty)\).
Weak localization correction (orthogonal metal class):

\[ \delta \sigma_{WL} = -\frac{8e^2}{h} D \int_{-\infty}^{\infty} d\eta \left\langle C^t_{\eta,-\eta}(x, x) \right\rangle_{\rho} = -\frac{e^2}{h} \frac{1}{\pi} \ln \left( \frac{\tau_{\phi}}{\tau_{el}} \right) \]

Even a good metal \((E_F \tau_{el} \gg 1)\) would localize at all temperatures in 2D without dephasing \((\tau_{\phi}(T) < \infty)\).

Isolated interacting, disordered system: ergodic phase must serve as its own heat bath (dephasing mechanism)

Differential equation for the Cooperon:

\[ \left\{ \partial_{\eta} - \frac{D}{2} \nabla^2 + \frac{i}{2} \left[ \rho_{cl} \left( x, t + \frac{\eta}{2} \right) - \rho_{cl} \left( x, t - \frac{\eta}{2} \right) \right] \right\} C^t_{\eta,\eta'}(x, x') = \frac{1}{2} \delta(\eta - \eta') \delta(x - x') \]

Dephasing is due to thermal fluctuations of the charge density \(\rho_{cl}(x, t)\).
Aside: Theory of the ergodic metal

- Field theory of the ergodic metal in $d > 1$: Finkel’stein nonlinear sigma model
  \[ Z = \int \mathcal{D}\rho \mathcal{D}\hat{\phi} e^{-S} \]
  \[ S = \int \operatorname{Tr} \left[ \frac{1}{4\lambda} (\nabla \hat{\phi})^2 + i\hbar (\hat{\omega} \hat{\phi}^3 \hat{\phi}) \right] - i\hbar \int \operatorname{Tr} \left[ \left( \rho_{\text{cl}} \hat{1} + \rho_{\text{q}} \hat{\gamma}^1 \right) \hat{M}_F \hat{q} \hat{M}_F \right] - i \frac{4}{\pi} \frac{\hbar}{\gamma} \int \rho_{\text{cl}} \rho_{\text{q}} \]

- Most physical formulation: Keldysh response theory

- Matrix part: order-by-order quantum interference corrections to conductance. One small parameter ($1/G \sim \lambda$).

- One-loop WL:

  Hydro part: Must average each diagram over the “bath” (hydro density fluctuations)!

Reviews:
- Kamenev, Levchenko, Adv Phys 2009
- Liao, Levchenko, MSF Ann Phys 2017

Finkel’stein 1983
• Field theory of the ergodic metal in $d > 1$:
  Finkel’stein nonlinear sigma model

• Most physical formulation: Keldysh response theory

$$Z = \int D\rho D\hat{q} e^{-S}$$

$$S = \int \text{Tr} \left[ \frac{1}{4\lambda} (\nabla \hat{q})^2 + i\hbar (\hat{\omega} \hat{\sigma}^3 \hat{q}) \right] - i\hbar \int \text{Tr} \left[ (\rho_{cl} \hat{1} + \rho_q \hat{\gamma}^1) \hat{M}_F \hat{q} \hat{M}_F \right] - i \frac{\hbar}{\gamma} \int \rho_{cl} \rho_q$$

• **Structurally analogous** (but we **know** the mother theory!):

The net influence each incoming string has on the other comes from adding together the influences involving diagrams with ever more loops.
MBL in 2D? Try to approach from the **ergodic (diffusive metal) side**

- **Differential equation for the Cooperon:**

  \[
  \begin{aligned}
  \left\{ \partial_\eta - \frac{D}{2} \nabla^2 + \frac{i}{2} \left[ \rho_{\text{cl}} \left( x, t + \frac{\eta}{2} \right) - \rho_{\text{cl}} \left( x, t - \frac{\eta}{2} \right) \right] \right\} C_{\eta,\eta'}^t (x, x') &= \frac{1}{2} \delta(\eta - \eta') \delta(x - x') \\
  \end{aligned}
  \]

- **Path integral representation**

  \[
  \langle C_{\eta,-\eta}^t (x, x) \rangle_\rho = \frac{1}{2} \int D\mathbf{r}(\tau) e^{-S}
  \]

  \[
  S = \int_{-\eta}^{\eta} d\tau \frac{1}{2D} \dot{\mathbf{r}}^2(\tau) + \frac{1}{4} \int_{-\eta}^{\eta} d\tau_1 \int_{-\eta}^{\eta} d\tau_2 
  \begin{bmatrix}
  \Delta \left( \mathbf{r}(\tau_1) - \mathbf{r}(\tau_2), \frac{\tau_1 - \tau_2}{2} \right) \\
  - \Delta \left( \mathbf{r}(\tau_1) - \mathbf{r}(-\tau_2), \frac{\tau_1 - \tau_2}{2} \right)
  \end{bmatrix}
  \]
Diffusion equation for the Cooperon:

\[
\begin{aligned}
\left\{ & \partial_{\eta} - \frac{D}{2} \nabla^2 + \frac{i}{2} \left[ \rho_{\text{cl}} \left( \mathbf{x}, t + \frac{\eta}{2} \right) - \rho_{\text{cl}} \left( \mathbf{x}, t - \frac{\eta}{2} \right) \right] \right\} 
C_{\eta,\eta'}^{t} \left( \mathbf{x}, \mathbf{x}' \right) = \frac{1}{2} \delta(\eta - \eta') \delta(\mathbf{x} - \mathbf{x}')
\end{aligned}
\]

Path integral representation

\[
\langle C_{\eta,-\eta}^{t} (\mathbf{x}, \mathbf{x}) \rangle_\rho = \frac{1}{2} \int \mathcal{D}\mathbf{r}(\tau) e^{-S}
\]

\[
S = \int_{-\eta}^{\eta} d\tau \frac{1}{2D} \dot{\mathbf{r}}^2(\tau) + \frac{1}{4} \int_{-\eta}^{\eta} d\tau_1 \int_{-\eta}^{\eta} d\tau_2 \begin{bmatrix}
\Delta \left( \mathbf{r}(\tau_1) - \mathbf{r}(\tau_2), \frac{\tau_1 - \tau_2}{2} \right) \\
- \Delta \left( \mathbf{r}(\tau_1) - \mathbf{r}(-\tau_2), \frac{\tau_1 - \tau_2}{2} \right)
\end{bmatrix}
\]

Dynamically screened long-range Coulomb interactions: Noise kernel is Markovian (bath is Ohmic):

\[
\Delta(\mathbf{k}, \omega) \simeq k_B T \frac{8\pi e^2}{Dq_{TF} k^2}
\]

Always dephases. Solve exactly; same as SCBA

Altshuler, Aronov, Khmelnitsky (1982)
MBL in 2D? Try to approach from the ergodic (diffusive metal) side

\[ S = \int_{-\eta}^{\eta} d\tau \frac{1}{2D} \dot{r}^2(\tau) + \frac{1}{4} \int_{-\eta}^{\eta} d\tau_1 \int_{-\eta}^{\eta} d\tau_2 \left[ \Delta \left( r(\tau_1) - r(\tau_2), \frac{\tau_1 - \tau_2}{2} \right) \right. \\
\left. - \Delta \left( r(\tau_1) - r(-\tau_2), \frac{\tau_1 - \tau_2}{2} \right) \right] \]

- Short-range interacting (\( \gamma \)), isolated system. Noise kernel is non-Markovian (diffusive!):

\[ \Delta(r, \tau) \simeq k_B T \frac{\gamma^2}{\kappa} \frac{1}{4\pi D_c |\tau|} \exp\left(-\frac{r^2}{4D_c |\tau|}\right) \]

- Does it always dephase? Exact solution??

- SCBA (2D):

\[ \frac{1}{\tau_\phi} = \frac{1}{2\pi \sigma_0} \frac{\gamma^2}{2 - \gamma} k_B T \ln (k_B T \tau_\phi) \]

Zala, Narozhny, Aleiner (2002)

Is it true? Or does dephasing fail at sufficiently low-\( T \) and/or for sufficiently small diffusion constant \( D \)?
Self-dephasing random walk

\[ S = \int_{-\eta}^{\eta} d\tau \frac{1}{2D} \dot{r}^2(\tau) + \frac{1}{4} \int_{-\eta}^{\eta} d\tau_1 \int_{-\eta}^{\eta} d\tau_2 \left[ \Delta \left( r(\tau_1) - r(\tau_2), \frac{\tau_1 - \tau_2}{2} \right) - \Delta \left( r(\tau_1) - r(-\tau_2), \frac{\tau_1 - \tau_2}{2} \right) \right] \]

- Non-Markovian action is similar to a self-avoiding random walk:

\[ S = \int d\tau \frac{1}{2} \dot{r}^2(\tau) + g \int d\tau_1 \int d\tau_2 \delta [r(\tau_1) - r(\tau_2)] \]
IDEA: View as geometric stat mech problem, look for scaling

- Finite dephasing length $L_\phi$ at large $T$ due to “self-interactions”
- Use RG to look for a new critical point at low but nonzero $T$
**Self-dephasing random walk**

- **Technique:** replicated path integral

**Cooperon:**
\[
\begin{align*}
 a, k, \omega & \xrightarrow{\text{replication}} a, k, \omega \\
= & \frac{1}{k^2 - i\hbar \omega + r}
\end{align*}
\]

**Thermal density fluctuation:**
\[
\begin{align*}
 k, \omega & \xrightarrow{\text{replication}} -k, -\omega \\
= & \frac{2k^2}{k^4 + \omega^2}
\end{align*}
\]

**Vertices:**
\[
\begin{align*}
 a, k + q, \omega + \Omega/2 & \xrightarrow{\text{interaction}} q, \Omega \\
 a, k, \omega & \xrightarrow{\text{interaction}} a, k + q, \omega - \Omega/2 \\
= & \mp \pi i \sqrt{g}
\end{align*}
\]

**Coupling strength (in \( d = 2 \)):**
\[
g = \frac{1}{4\pi^2} \frac{k_B T \gamma^2}{\kappa D_c}
\]

**Dimensional analysis (\( z = 2, d \) dimensions):**
\[
[g] = 4 - d \equiv \epsilon
\]
• **Technique:** replicated path integral

Cooperon: \[ a, k, \omega \rightarrow a, k, \omega = \frac{1}{k^2 - i\hbar \omega + r} \]

We pin the Cooperon diffusion constant with a wave function renormalization (why not—we have to pin something)

**Logic:**

• The Cooperon correction is the virtual shift of the diffusion constant

• Diffusion constant itself should not run due to dephasing (non-self-consistent problem, real vs. virtual processes)

• Big question: Can we implement self-consistency in a controlled (or at least plausibly believable) way??
Self-dephasing random walk

\[ \frac{dg}{dl} = \epsilon g - g^2 \frac{4}{(h + 2)^3}, \]
\[ \frac{dr}{dl} = r \left[ 2 - g \frac{2}{(h + 2)^3} \right] + \frac{g}{2} \frac{1}{(h + 2)}, \]
\[ \frac{dh}{dl} = -g \frac{2}{(h + 2)^3} h. \]

- **Non-trivial fixed point:** \( g^* = 2\epsilon, \ r^* = -\frac{\epsilon}{4}, \ h^* = 0 \)

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Self-dephasing random walk

\[
\frac{dg}{dl} = \epsilon g - g^2 \frac{4}{(h + 2)^3}, \\
\frac{dr}{dl} = r \left[ 2 - g \frac{2}{(h + 2)^3} \right] + g \frac{1}{2(h + 2)}, \\
\frac{dh}{dl} = -g \frac{2}{(h + 2)^3} h
\]

Non-trivial fixed point: \( g^* = 2\epsilon, \quad r^* = -\frac{\epsilon}{4}, \quad h^* = 0 \)

"Order parameter/mass": Dephasing rate

\[
\frac{1}{\tau_\phi} \equiv \delta r = r - r^* \to 0
\]

Correlation length exponent

\[
\delta r \sim \left( \frac{L}{a} \right)^{1/\nu}, \quad \nu = \frac{2}{d} = \frac{2}{4 - \epsilon}
\]

Chayes bound \( \nu \geq 2/d \)

Ergodic Metal
Cooperon at the non-trivial fixed point:

- **Scaling dimension:**
  \[ d_c(l) = -2 - \frac{g_h(l)}{[h(l) + 2]^3} \rightarrow -2 \]

- **Scaling form:**
  \[ \langle C \rangle \rho(k, \omega = 0) = \frac{1}{Dk^2} \]

- **Weak localization correction:**
  \[
  \delta \sigma_{WL} = -\frac{8e^2}{h} \frac{D}{2} \int d^2k \frac{1}{Dk^2} = -\frac{e^2}{h} \frac{1}{\pi} \ln \left( \frac{\tau_\phi \rightarrow \infty}{\tau_{el}} \right) \rightarrow -\infty \quad \text{at} \quad \frac{k_B T^*}{\kappa D_c} \gamma^2 \sim \epsilon
  \]

Failure of dephasing at finite temperature \( T^* \): “toy” MBL transition!

- **Does it survive to** \( \epsilon = 2 \) ?
- **WL is first correction. Must analyze “AAK” problem at each order**
- **Self-consistent (“RG-improved PT”) solution: running \( D \)**

Liao and MSF 1710.05037
2. Geometric percolation at the surface of a topological superconductor

Sayed Ali Akbar Ghorashi, Yunxiang Liao, MSF

1. Dephasing story:
   Liao, MSF arXiv: 1710.05037

2. Percolation story:
   Ghorashi, Liao, MSF arXiv: 1711.03972
A. **Anderson localized plateau**  
**(topological Anderson insulator)**

\[ \sigma_{xx} = 0, \]

\[ \sigma_{xy} = \frac{\nu e^2}{h} \]

B. **Plateau transition ~ percolation**  
**(critical delocalization / “MIT”)**

\[ \sigma_{xx} \sim \frac{e^2}{2h} \]

C. **Anderson localized plateau**

\[ \sigma_{xx} = 0, \]

\[ \sigma_{xy} = \frac{(\nu+1)e^2}{h} \]
A. Anderson localized plateau (topological Anderson insulator)

\[ \sigma_{xx} = 0, \]
\[ \sigma_{xy} = \frac{\nu e^2}{h} \]

- Bulk states are localized
- **Chiral edge states delocalized (conduct current) at all energies in plateau!**
• Integer QHE not equivalent to classical percolation

• Different kind of integer quantum Hall effect IS described by 2D classical percolation: Spin Quantum Hall effect

\[ \sigma_{xx}^s = 0, \]
\[ \sigma_{xy}^s = \frac{2\nu q^2}{\hbar}, \quad q = \frac{\hbar}{2} \]

= a quantum Hall effect for spin (conserved! No SOC)

• Realizations? Gapless quasiparticles in a superconductor (SC)
**Superconductivity**

- Superfluidity: Electrical resistance is zero
- No heat or spin transport in the superfluid

**3D Bulk topological superconductivity**

Integer-valued winding number \( \nu \in \mathbb{Z} \)

2D Majorana (DIII) or Dirac (CI, AIII) surface fluid
- Heat and spin transport?
- Stability? (Exposed crystal surface: disorder)

**Experimental realizations**
- Helium 3B (neutral topological superfluid)
- No materials please, we’re theorists
**Superconductivity**

- **Superfluidity**: Electrical resistance is zero
- **No heat or spin transport in the superfluid

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**Class CI: Time-reversal, spin SU(2) symmetries**

**Dirac surface fluid, incorporating TRI disorder**

D. Bernard (1997); Mudry, Chamon, Wen (1996); Caux, Kogan, Tsvelik (1996)
Ludwig (2000); Bhaveen, Caux, Kogan, Tsvelik (2001)
Ostrovsky, Gornyi, Mirlin (2006); MSF and Yuzbashyan (2012)

**Even winding numbers only** $\nu = 2k$

\[
\hat{h} = \begin{bmatrix}
0 & (-i\partial)^k + (A_x^a + iA_y^a)(\mathbf{r}) \hat{r}^a \\
(-i\partial)^k + (A_x^a - iA_y^a)(\mathbf{r}) \hat{r}^a & 0
\end{bmatrix}
\]
Multifractal analysis: universal wavefunction probability statistics

Multifractal spectrum

- Box probability: \( \mu_i = \int \mathrm{d}^2 \mathbf{r} |\psi(\mathbf{r})|^2 \)
- IPR: \( P_q = \sum_i \mu_i^q \)
- Moment spectrum: \( P_q \sim (b/L)^{\tau(q)} \)
  \[ \tau(q) = (q - 1)(2 - \theta q), \quad |q| \leq q_c = \sqrt{2}/\theta \]

Multifractal: fractal level sets

- Large q: scaling of peaks
  \( \tau(q) = \alpha_- q, \quad q > q_c \)
- Large negative q: scaling of valleys
  \( \tau(q) = \alpha_+ q, \quad q < -q_c \)
  \[ \alpha_\mp = (\sqrt{2} \mp \sqrt{\theta})^2 \]
Multifractal analysis: universal wavefunction probability statistics

**Multifractal spectrum**
- Box probability: \( \mu_i = \int d^2r \, |\psi(r)|^2 \)
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\[ \tau(q) = (q - 1)(2 - \theta q), \quad |q| \leq q_c = \sqrt{2/\theta} \]

**Universal predictions**
- **Low-energy surface states, topological superconductor (TSC) (class CI)**
  \[ \theta_k = 1/2(k + 1), \quad Sp(2n)_k \]
- **Percolation (Spin quantum Hall plateau transition, class C)**
  \[ \theta \sim 1/8 \]

- **Orthogonal metal class:** \( \tau(q) = 0, \; \text{(localized)} \)

*Topological protection from localization at finite energy?*

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Mudry, Chamon, Wen (1996)
Caux, Kogan, Tsvelik (1996)
MSF and Yuzbashyan (2012)

Evers, Mildenberger, Mirlin (2003)
Essin and Guraire (2015)
Multifractal analysis: universal wavefunction probability statistics

Multifractal spectrum

- Box probability: \( \mu_i = \int \mathcal{d}^2 \vec{r} \, |\psi(\vec{r})|^2 \)

- IPR: \( P_q = \sum_i \mu_i^q \)

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\[ \tau(q) = (q - 1)(2 - \theta q), \quad |q| \leq q_c = \sqrt{2/\theta} \]

Symmetry (sigma model) argument? Not really...

Class CI surface states: Energy (frequency) breaks symmetry down...

\[ G \times G \to G, \quad G = \text{Sp}(4n) \]

to Class C?

\[ G = \frac{\text{Sp}(4n)}{\text{U}(2n)}; \quad \text{WZNW} \Rightarrow \text{Pruisken theta term} \]

to orthogonal class?

\[ G = \frac{\text{Sp}(4n)}{\text{Sp}(2n) \times \text{Sp}(2n)} \]

Usually: CI to orthogonal metal, but

1. Surface states are bound (energy roof)
2. Effect of WZNW on RG flow
Numerical results: low, finite-energy multifractal statistics

\[ \Delta(q) \equiv \tau(q) - 2(q - 1) \]

\( k = 1 \)  \( k = 7 \)  \( k = 8 \)  \( k = 1 \)

(a) Numerical
\( LE, q_c(k=1)=2\sqrt{2} \)
\( FE, q_c(C)=4 \)

(b) Numerical
\( LE, q_c(k=1)=2\sqrt{2} \)
\( FE, q_c(C)=4 \)

(c) Numerical
\( LE, q_c(k=7)=2\sqrt{8} \)
\( FE, q_c(C)=4 \)

(d) Numerical
\( LE, q_c(k=7)=2\sqrt{8} \)
\( FE, q_c(C)=4 \)

(e) Numerical
\( LE, q_c(k=8)=6 \)
\( FE, q_c(C)=4 \)

(f) Numerical
\( LE, q_c(k=8)=6 \)
\( FE, q_c(C)=4 \)

(g) Numerical
\( LE, q_c(k=1)=2\sqrt{2} \)
\( FE, q_c(C)=4 \)

(a,c,e): low-energy states

(b,d,f): finite-energy states

Low energy states, but with
- Broken time-reversal (random mass)
- Preserved spin SU(2)
- Zero average mass
- Should be SQHP transition!!

Ghorashi, Liao, MSF 1711.03972
Numerical results: Population statistics

(Arbitrary) fitness criteria: Compare analytical and numerical

$$\text{error}(q) = \frac{|\tau_N(q) - \tau_A(q)|}{|\tau_A(q)|}$$

If error is less than 6% for 75% or more of the q-points in the interval $0 \leq q \leq q_c$, keep that state.

Ghorashi, Liao, MSF 1711.03972
A. The dry season: walk (drive in Houston)

B. Tropical rainstorm floods to the percolation threshold (every April or May in Houston): drive or boat

C. Hurricane Harvey: boat
Surface states of topological superconductors with quenched disorder (class CI):

Percolation/SQHP states (almost) all the way down!

Ghorashi, Liao, MSF 1711.03972

Other classes:
AllI TRI, spin U(1) symmetry

- Winding number $\nu = \frac{1}{2}$: IQHP states (almost) all the way down
  - Ostrovsky, Gornyi, Mirlin (2007)
  - Chou and Foster (2014)

- Higher $\nu$? Conjecture: IQHP states (almost) all the way down.

DIII TRI, no spin symmetry (SOC)
...in progress...
Dephasing catastrophe in $4 - \varepsilon$ dimensions due to diffusive (non-Markovian) bath: a toy ergodic-MBL transition

Q: Does the transition survive to $d = 2$ ($\varepsilon = 2$)?
Q: Test with a lattice polymer simulation
Q: Non-perturbative (CFT?) approach…
Surface states of topological superconductors with quenched disorder (class CI):

- Percolation/SQHP states (almost) all the way down!
- Most states at the surface of a TRI topological superconductor are plateau transition states of the corresponding quantum Hall effect!

Q: Generalization to AIII ↔ A (“ordinary” QHE) and DIII ↔ D (thermal QHE)?

Q: FQH Plateau transitions ↔ fractionalized surface fluid?
An analytical argument that doesn’t work:

Non-linear sigma model for the class CI surface states at zero energy:

\[
S = \frac{\pi}{8} \sigma_{xx} \int d^2 \mathbf{r} \, \text{Tr} \left[ \nabla \hat{q}^\dagger \cdot \nabla \hat{q} \right] - i \frac{k}{12\pi} \int d^3 \mathbf{r} \, \epsilon^{ijk} \, \text{Tr} \left[ (\hat{q}^\dagger \partial_i \hat{q})(\hat{q}^\dagger \partial_j \hat{q})(\hat{q}^\dagger \partial_k \hat{q}) \right]
\]

Numerics: finite energy states are class C.

Impose class C constraint: \( \hat{q}^\dagger = \hat{q} \)

Wess-Zumino-Novikov-Witten term becomes theta term (SQHP!)

Ala Bocquet, Serban, Zirnbauer (2000)

\[
S \to \frac{\pi}{8} \sigma_{xx} \int d^2 \mathbf{r} \, \text{Tr} \left[ \nabla \hat{q} \cdot \nabla \hat{q} \right] - i (\pi k) \frac{1}{16\pi i} \int d^2 \mathbf{r} \, \epsilon^{ij} \, \text{Tr} \left[ \hat{q} \partial_i \hat{q} \partial_j \hat{q} \right]
\]

Suggests \( k = \text{odd (even) different (?) (plateau vs plateau transition)} \)

[? or always localized: Essin and Gurarie (2015)]

However, numerical results show no even/odd effect

Always delocalized, with SQHP statistics