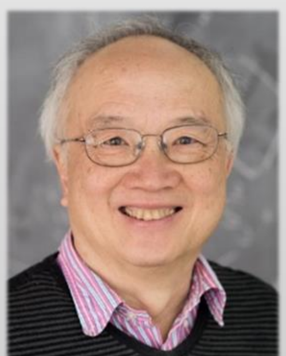
Interference-mediated pairing in dirty marginal Fermi liquids

Matthew S. Foster
Rice University

December 16, 2023



Tsz Chun Wu
Rice University, Trexquant

Patrick Lee
MIT

- **Wu, Lee, Foster**
  PRB **108**, 214506 (2023)

Related:

- **Quantum hydrodynamics of a dirty MFL**
  Wu, Liao, Foster (PRB 2022)

- **Shot noise suppression in a dirty MFL**
  Wu, Foster arXiv:2312.03071

Outline

I. **Review:** BCS pairing of fractal wave functions

II. **Example:** Amplitude and stiffness in the Aubry-Andre (Hofstadter Butterfly) model

III. **Interference-mediated pairing** in a dirty marginal Fermi liquid

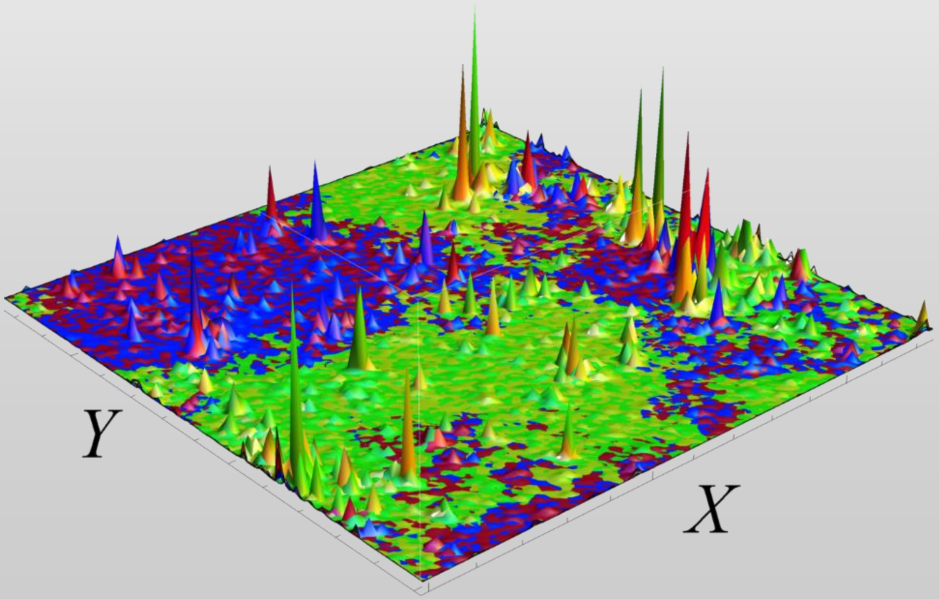
Enhanced spatial correlations near the Anderson MIT

Chalker-scaling correlator (energy-split IPR)

$$C_{\varepsilon_1,\varepsilon_2} \equiv \frac{\sum_{\mathbf{r}} |\psi_{\varepsilon_1}(\mathbf{r})|^2 |\psi_{\varepsilon_2}(\mathbf{r})|^2}{\frac{1}{2}\left(\sum_{\mathbf{r}} |\psi_{\varepsilon_1}(\mathbf{r})|^4 + \sum_{\mathbf{r}} |\psi_{\varepsilon_2}(\mathbf{r})|^4\right)}$$

Chalker and Daniell 1988 Chalker 1990

Near a single-particle Anderson MIT, states nearby in energy "clump" at rare peaks in position space



Enhanced spatial correlations near the Anderson MIT

Chalker-scaling correlator (energy-split IPR)

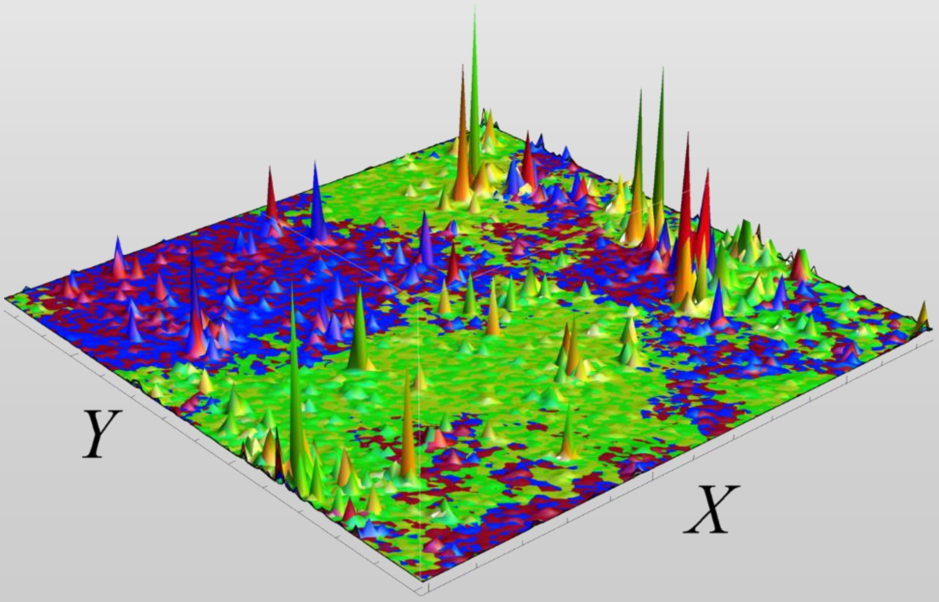
$$C_{\varepsilon_1,\varepsilon_2} \equiv \frac{\sum_{\mathbf{r}} |\psi_{\varepsilon_1}(\mathbf{r})|^2 |\psi_{\varepsilon_2}(\mathbf{r})|^2}{\frac{1}{2}\left(\sum_{\mathbf{r}} |\psi_{\varepsilon_1}(\mathbf{r})|^4 + \sum_{\mathbf{r}} |\psi_{\varepsilon_2}(\mathbf{r})|^4\right)}$$

Chalker and Daniell 1988 Chalker 1990

Near a single-particle Anderson MIT, states nearby in energy "clump" at rare peaks in position space

$$C_{\varepsilon_1,\varepsilon_2} \sim |\varepsilon_1 - \varepsilon_2|^{-\frac{d-\tau_2}{d}}$$

$$0 < \tau_2 < d$$



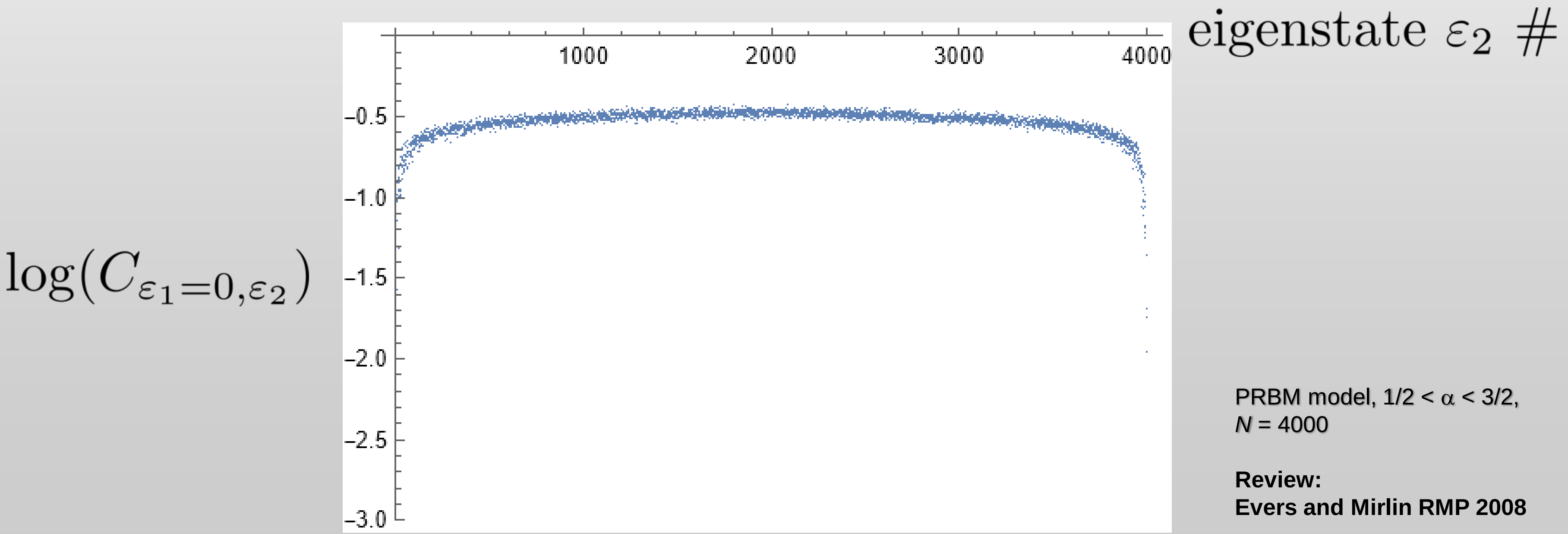
Enhanced spatial correlations near the Anderson MIT

Chalker-scaling correlator (energy-split IPR)

$$C_{\varepsilon_1,\varepsilon_2} \equiv \frac{\sum_{\mathbf{r}} |\psi_{\varepsilon_1}(\mathbf{r})|^2 |\psi_{\varepsilon_2}(\mathbf{r})|^2}{\frac{1}{2}\left(\sum_{\mathbf{r}} |\psi_{\varepsilon_1}(\mathbf{r})|^4 + \sum_{\mathbf{r}} |\psi_{\varepsilon_2}(\mathbf{r})|^4\right)}$$

Chalker and Daniell 1988 Chalker 1990

Near a single-particle Anderson MIT, states nearby in energy "clump" at rare peaks in position space



PRBM model, 1/2 < α < 3/2, *N* = 4000

Review:
Evers and Mirlin RMP 2008

Enhanced spatial correlations near the Anderson MIT

Chalker-scaling correlator (energy-split IPR)

$$C_{\varepsilon_1,\varepsilon_2} \equiv \frac{\sum_{\mathbf{r}} |\psi_{\varepsilon_1}(\mathbf{r})|^2 |\psi_{\varepsilon_2}(\mathbf{r})|^2}{\frac{1}{2}\left(\sum_{\mathbf{r}} |\psi_{\varepsilon_1}(\mathbf{r})|^4 + \sum_{\mathbf{r}} |\psi_{\varepsilon_2}(\mathbf{r})|^4\right)}$$

Chalker and Daniell 1988 Chalker 1990

Near a single-particle Anderson MIT, states nearby in energy "clump" at rare peaks in position space

- **"Pairing of exact eigenstates"**
- **Can enhance pairing amplitude near the SIT**




Feigel'man, Ioffe, Kravtsov, Yuzbashyan 2007
Feigel'man, Ioffe, Kravtsov, Cuevas 2010

Enhanced spatial correlations near the Anderson MIT

Chalker-scaling correlator (energy-split IPR)

$$C_{\varepsilon_1,\varepsilon_2} \sim |\varepsilon_1 - \varepsilon_2|^{-\frac{d-\tau_2}{d}}, \qquad 0 < \tau_2 < d$$

Chalker and Daniell 1988 Chalker 1990

Near a single-particle Anderson MIT, states nearby in energy "clump" at rare peaks in position space

$$\Delta_{\varepsilon_1} = W \sum_{\varepsilon_2} C_{\varepsilon_1,\varepsilon_2} \tanh\left(\frac{E_{\varepsilon_2}}{2T}\right) \frac{\Delta_{\varepsilon_2}}{2E_{\varepsilon_2}}$$

$$\overline{\Delta}(T=0) \sim (\nu_0 W)^{d/(d-\tau_2)}$$

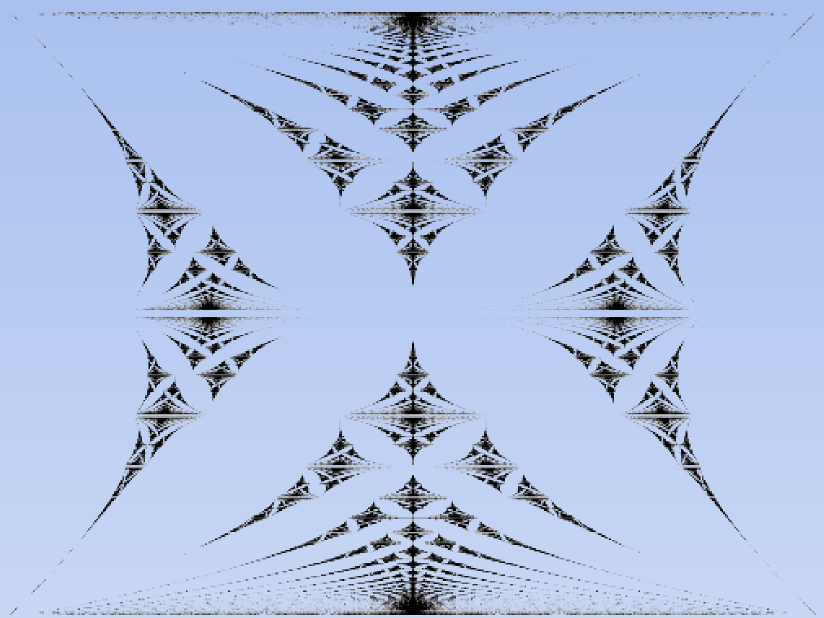
Feigel'man, Ioffe, Kravtsov, Yuzbashyan 2007
Feigel'man, Ioffe, Kravtsov, Cuevas 2010




Fractality and superconductivity: Numerical test

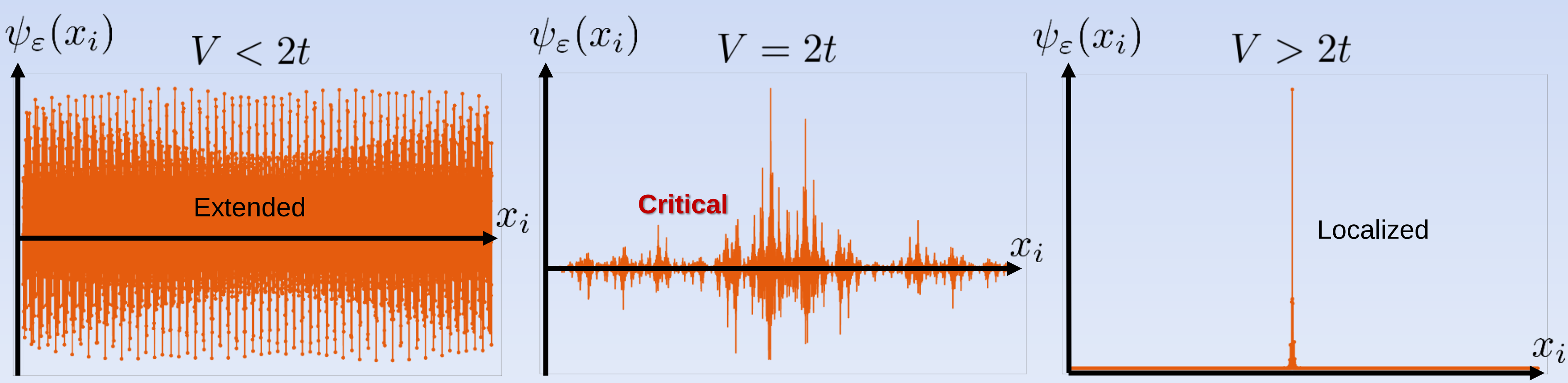
- **Aubry-Andre model: Uniform hopping in an incommensurate periodic potential**

$$H = -t\sum_{i\sigma}\left(c^\dagger_{i\sigma}c_{i+1\sigma} + c^\dagger_{i+1\sigma}c_{i\sigma}\right) + \sum_i\left[V\cos\left(2\pi\frac{p}{q}x_i\right) - \mu\right]n_i$$



- **Extended, localized, critical phases**

- **Critical phase (with Hofstadter butterfly spectrum)**

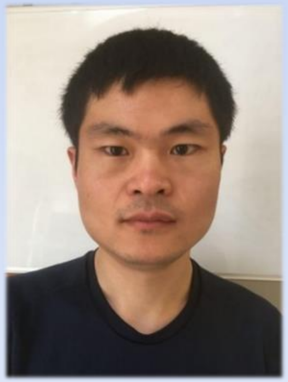


Fractality and superconductivity: Numerical test

- **Self-consistent static mean-field (BdG) numerics for the Aubry-Andre-Hubbard model**

$$H = -t \sum_{i\sigma} \left(c^\dagger_{i\sigma} c_{i+1\sigma} + \text{H.c.} \right) + \sum_i \left[V \cos\left(2\pi \frac{p}{q} i \right) - \mu \right] n_i - U \sum_i n_{i\uparrow} n_{i\downarrow}$$

  - **Same method as Ghosal, Randeria, Trivedi (1998)**



Xinghai Zhang

Rice University

- **Zhang and Foster** (PRB 2022)

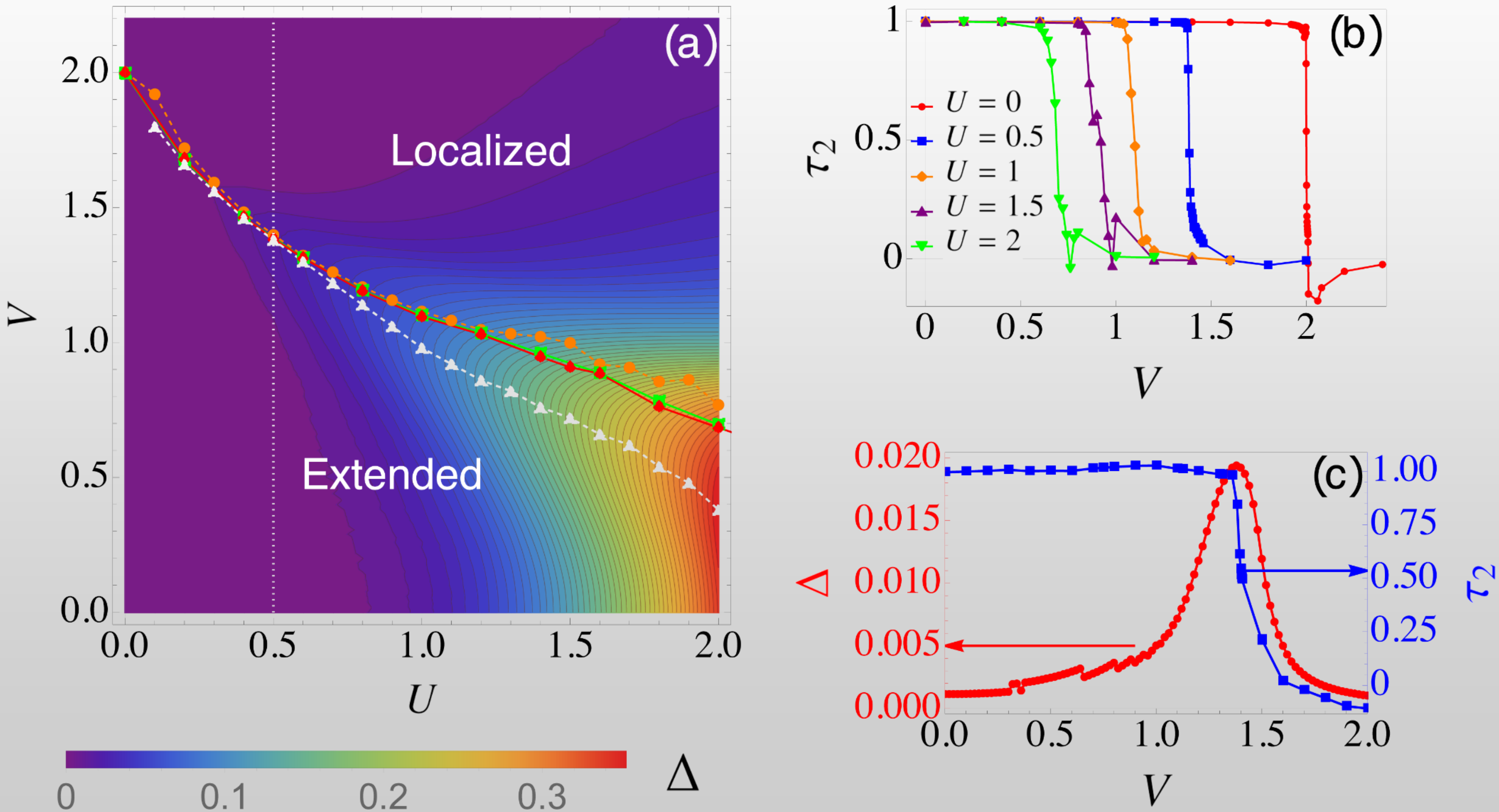
Related:

- **Anderson-Mott transition and spin glass order**
  Zhang and Foster 2309.13114

Early evidence of enhancement:
Fan, Chern, Lin 2021

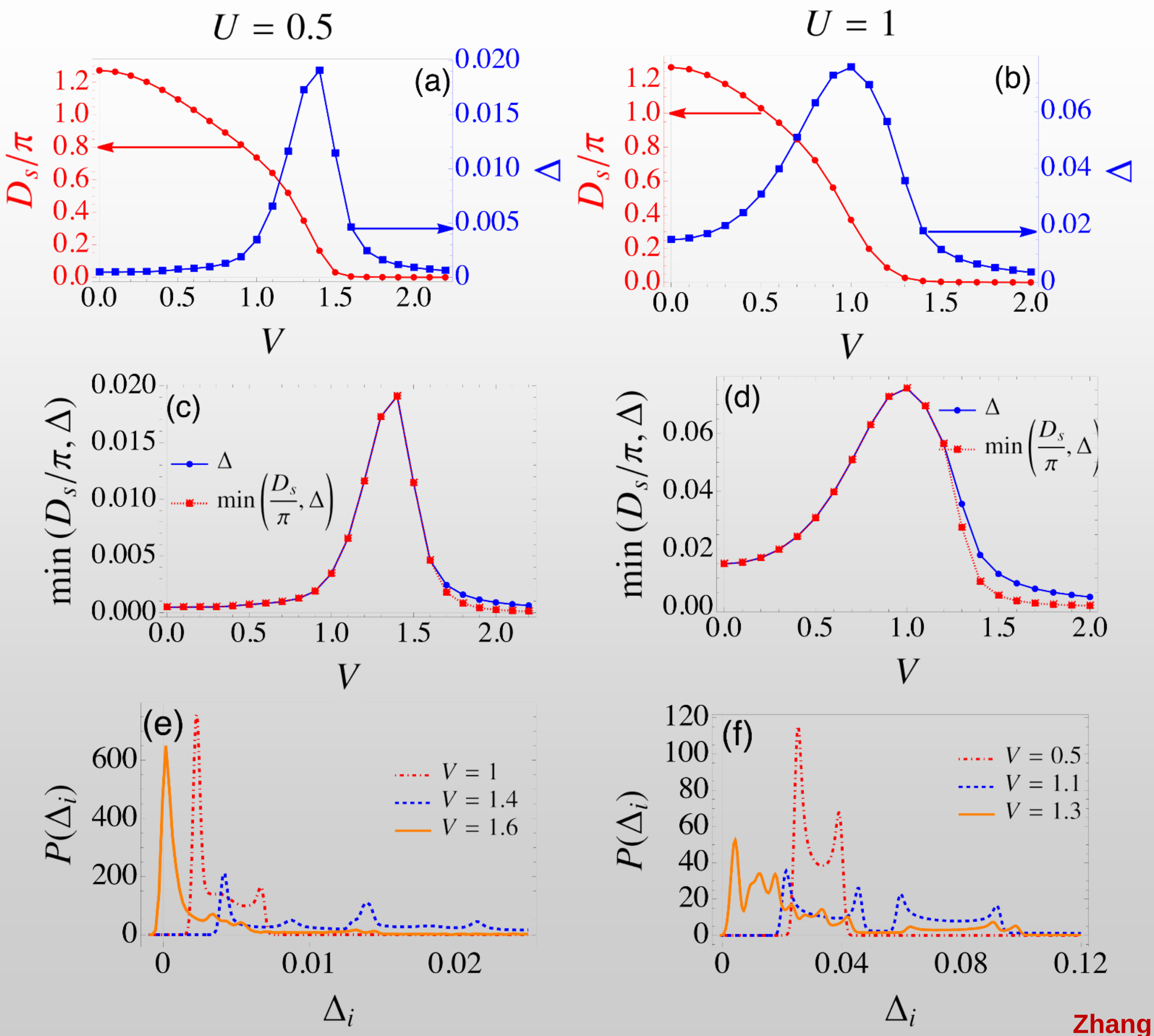
Fractality enhances superconductivity

- **Numerics: Renormalized single-particle MIT**
- ***BCS pairing strongly enhanced at the transition***



Zhang and Foster 2022

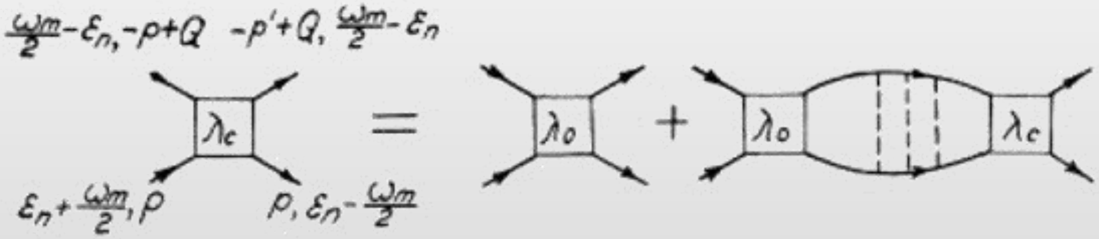
Amplitude and stiffness



Zhang and Foster 2022

Disorder in s-wave superconductors

- **Anderson's theorem (1960)**
  - **s-wave superconductivity is immune to non-magnetic disorder**
  - **T_c remains unchanged**



$$T_c \sim e^{-1/\nu_0 W}$$

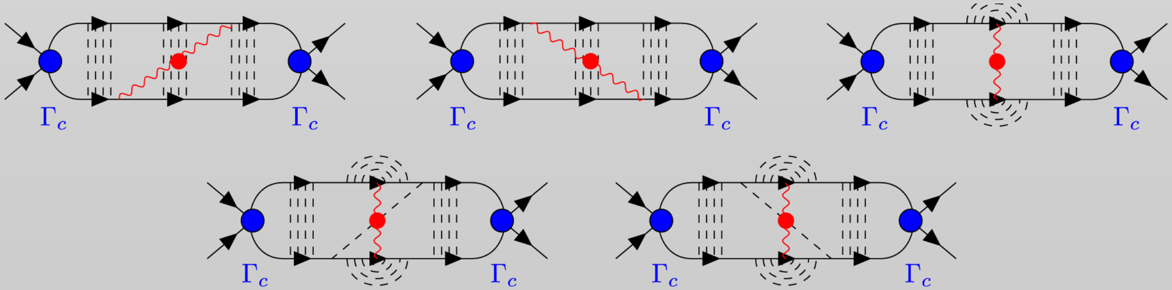
Review:
Altshuler and Aronov 1985

Disorder in s-wave superconductors

- **Anderson's theorem (1960)**
  - s-wave superconductivity is immune to non-magnetic disorder
  - T_c remains unchanged

- **Maekawa & Fukuyama (1982)**

How about Anderson localization?

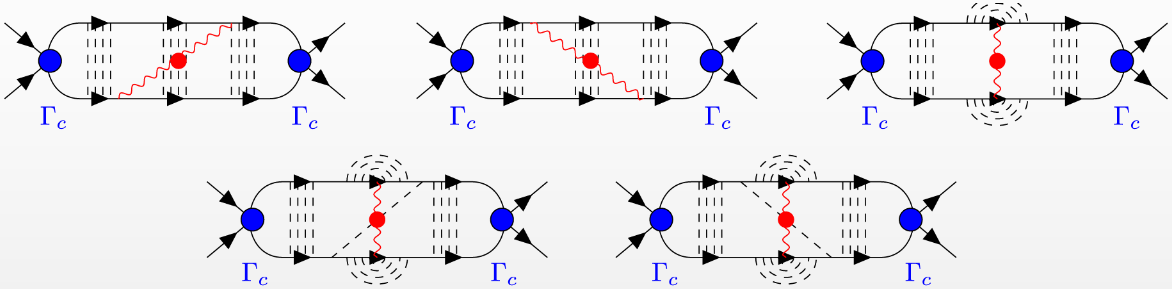


$$\frac{\delta T_c}{T_c} \sim -\ln^3\left(\frac{\Lambda}{T_c}\right)$$

Maekawa and Fukuyama 1982
Finkel'stein 1987

Multifractal enhancement in s-wave superconductors

- **Maekawa & Fukuyama (1982)**



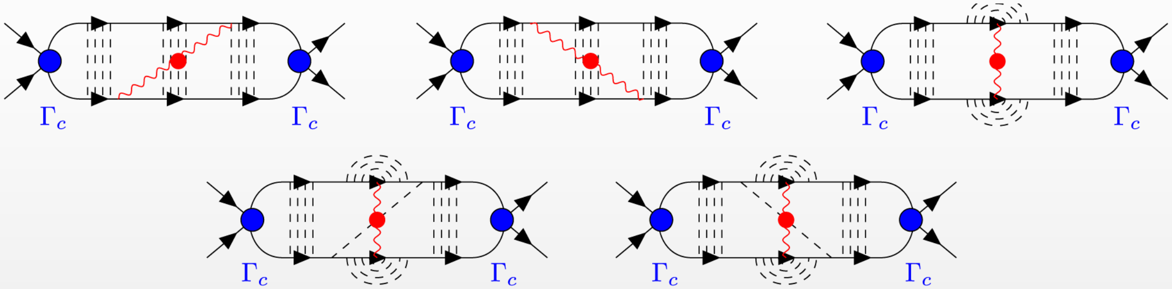
$$\frac{\delta T_c}{T_c} \sim -\ln^3\left(\frac{\Lambda}{T_c}\right)$$

Maekawa and Fukuyama 1982
Finkel'stein 1987

  - **Suppression due to quantum interference and long-ranged Coulomb interactions (SIT precursor)**

Multifractal enhancement in s-wave superconductors

- **Maekawa & Fukuyama (1982)**

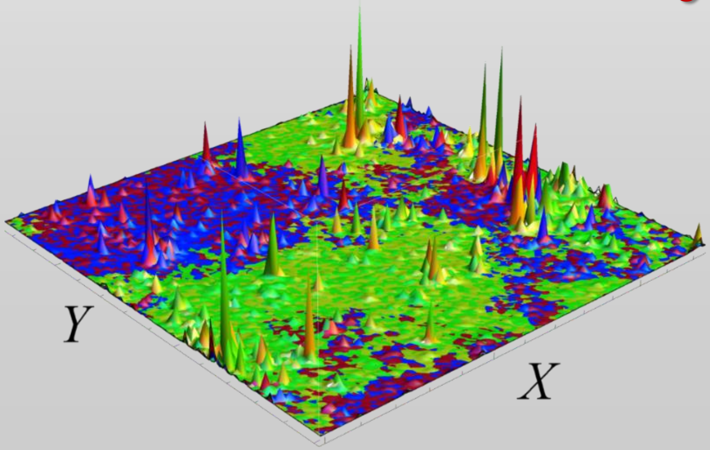


$$\frac{\delta T_c}{T_c} \sim -\ln^3\left(\frac{\Lambda}{T_c}\right)$$

Maekawa and Fukuyama 1982
Finkel'stein 1987

  - **Suppression due to quantum interference and long-ranged Coulomb interactions (SIT precursor)**

- ***Short-ranged,*** **other interactions + Chalker scaling:**
  Enhancement of T_c near Anderson MIT (2007)



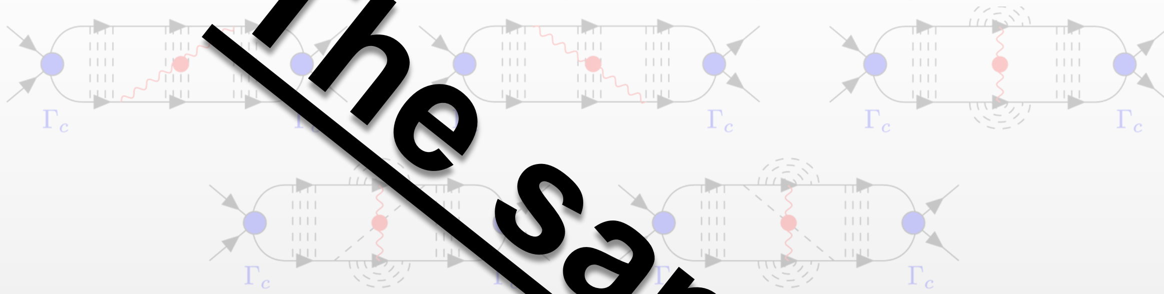
$$T_c \sim \frac{1}{\nu_0}\left(W\nu_0\right)^{\frac{d}{d-\tau_2}}$$

Feigel'man, Ioffe, Kravtsov, Yuzbashyan 2007
Feigel'man, Ioffe, Kravtsov, Cuevas 2010
Burmistrov, Gornyi, Mirlin 2012, 2015
Mayoh and Garcia-Garcia 2015
Fan and Garcia-Garcia 2020
Fan, Chern, Lin 2021
Stosiek, Evers, Burmistrov 2021

Multifractal enhancement in s-wave superconductors

The same mechanism!

- **Maekawa & Fukuyama (1982)**

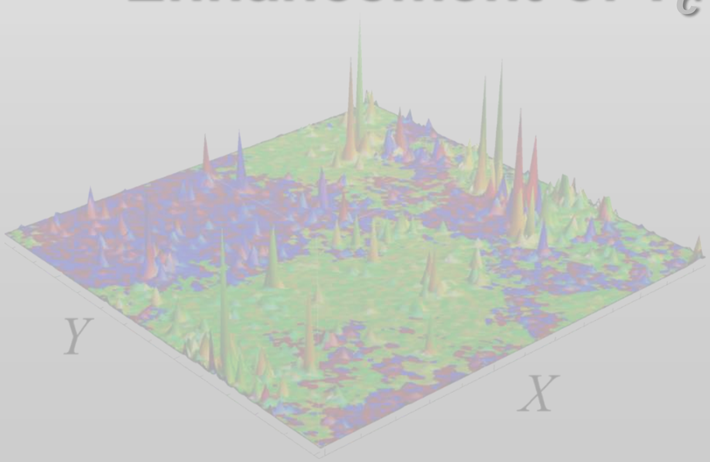


$$\frac{\delta T_c}{T_c} \sim -\ln^3\left(\frac{\Lambda}{T_c}\right)$$

Maekawa and Fukuyama 1982
Finkel'stein 1987

  - **Suppression due to quantum interference and long-ranged Coulomb interactions (SIT precursor)**

- ***Short-ranged,*** **other interactions + Chalker scaling: Enhancement of T_c near Anderson MIT (2007)**



$$T_c \sim \frac{1}{\nu_0}\left(W\nu_0\right)^{\frac{d}{d-d_2}}$$

Feigel'man, Ioffe, Kravtsov, Yuzbashyan 2007
Feigel'man, Ioffe, Kravtsov, Cuevas 2010
Burmistrov, Gornyi, Mirlin 2012, 2015
Mayoh and Garcia-Garcia 2015
Fan and Garcia-Garcia 2020
Fan, Chern, Lin 2021
Stosiek, Evers, Burmistrov 2021

Interference-mediated pairing in dirty marginal Fermi liquids

- **Wu, Lee, Foster** PRB **108**, 214506 (2023)

Tsz Chun Wu

Rice University, Trexquant

Patrick Lee

MIT

Model: Fermions, quantum-critical bosons, potential disorder

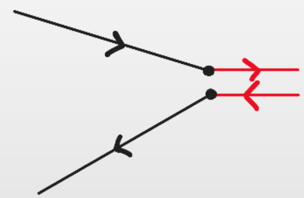
Ingredients:

- *N*-flavored electrons

- SU(*N*) matrix bosons

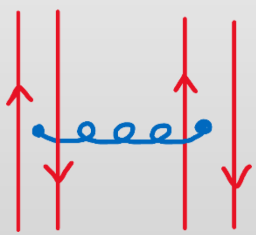
$$= \frac{1}{\omega^2 - \mathbf{k}^2}$$

- Yukawa coupling (e.g. FM)



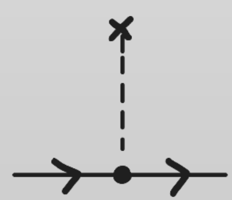
$$= \frac{g}{\sqrt{N}}$$

- Boson-boson interactions



$$= \frac{\lambda_\phi}{N^2}$$

- Potential disorder



Damia, Kachru, Raghu, Torroba 2019

Nosov, Burmistrov, Raghu 2020

Disorder-smeared saddle-point: MFL, quantum relaxational bosons

Self-consistent saddle-point solution at $N = \infty$, finite temperature T:

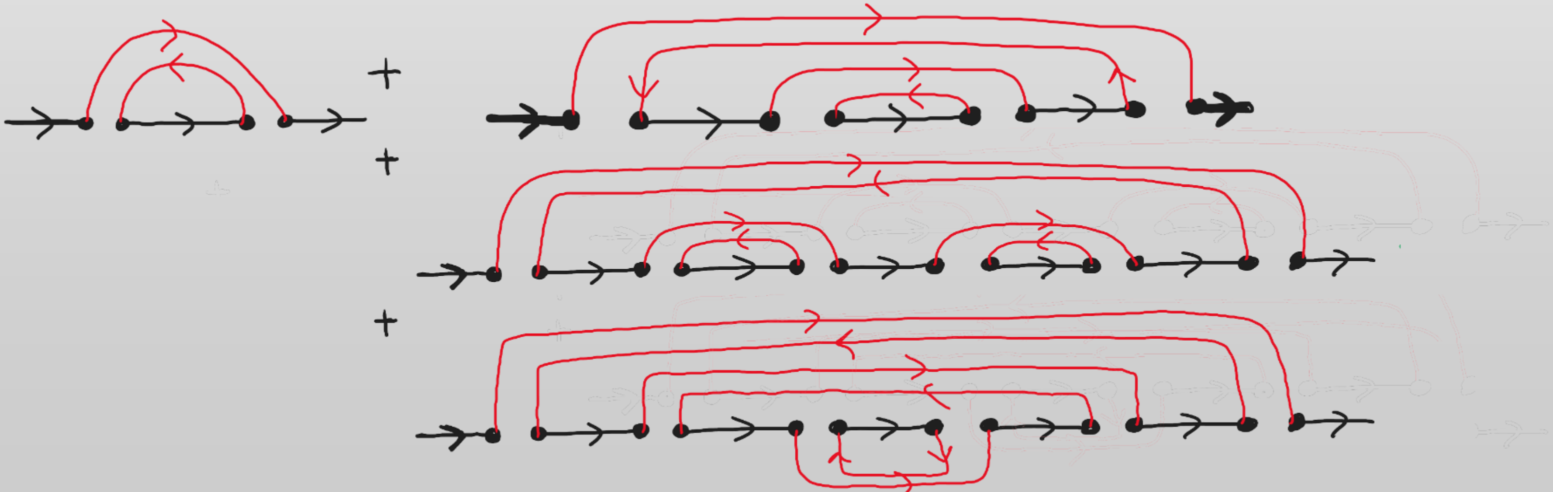
1. **Fermions: Impurity scattering, inelastic MFL**

$$\Sigma_R(\omega) = -i\gamma_{\text{el}} - \bar{g}^2 \left[\omega \ln\left(\frac{\omega_c}{x}\right) + i\frac{\pi}{2}x\right] \qquad \bar{g}^2 = \frac{g^2}{(2\pi)^2\gamma_{\text{el}}}$$

$$x = \max\left(\omega, J\,T\right), \quad J = J(\alpha_m/\alpha)$$

Patel, Guo, Esterlis, Sachdev 2023
Wu, Liao, Foster 2022

MFL from SCBA, using dressed (quantum relaxational) boson



Disorder-smeared saddle-point: MFL, quantum relaxational bosons

Self-consistent saddle-point solution at $N = \infty$, finite temperature T:

1. Fermions: Impurity scattering, inelastic MFL

$$\Sigma_R(\omega) = -i\gamma_{\text{el}} - \bar{g}^2 \left[\omega \ln\left(\frac{\omega_c}{x}\right) + i\frac{\pi}{2}x\right] \qquad \bar{g}^2 = \frac{g^2}{(2\pi)^2\gamma_{\text{el}}}$$

$$x = \max\left(\omega, J\,T\right), \quad J = J(\alpha_m/\alpha)$$

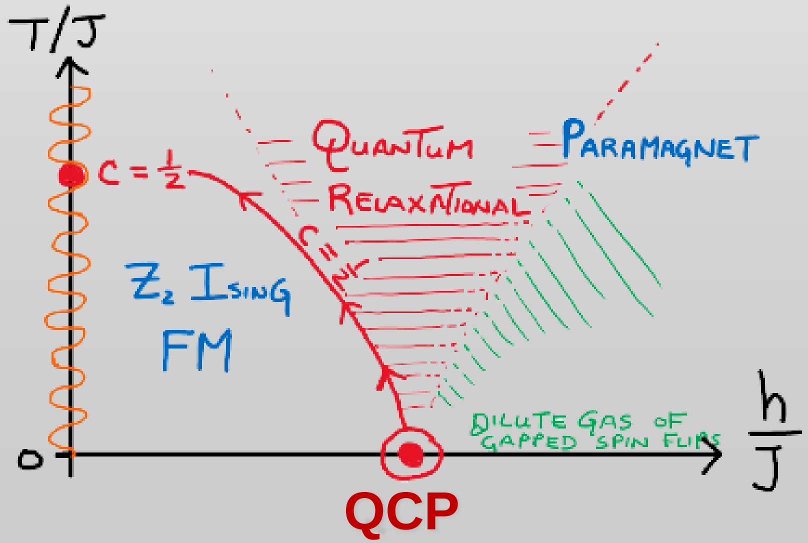
2. Bosons: Quantum relaxational

- Diffusive dynamics ($z = 2$)
- Thermal mass (boson-boson interactions)

Patel, Guo, Esterlis, Sachdev 2023
Wu, Liao, Foster 2022

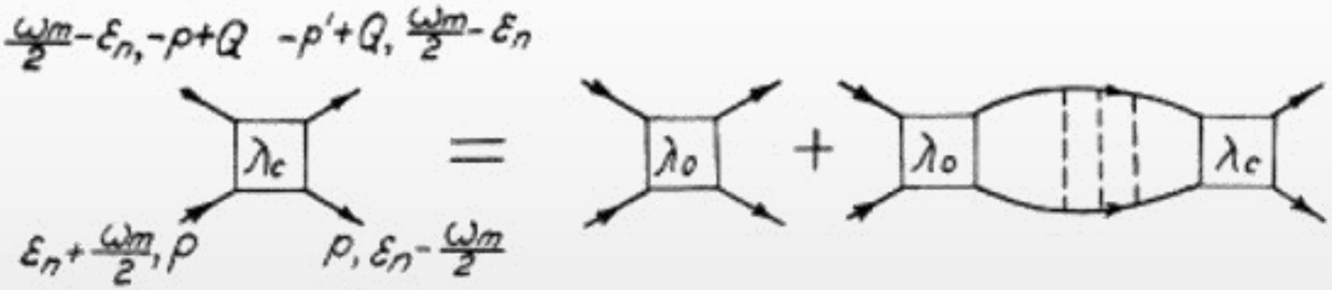
$$D_R(\omega, \mathbf{k}) = -\frac{1}{\mathbf{k}^2 - i\,\alpha\,\omega + \alpha_m\,T}$$

- *Generic form above a QCP*



Pairing in the MFL 1: Semiclassics

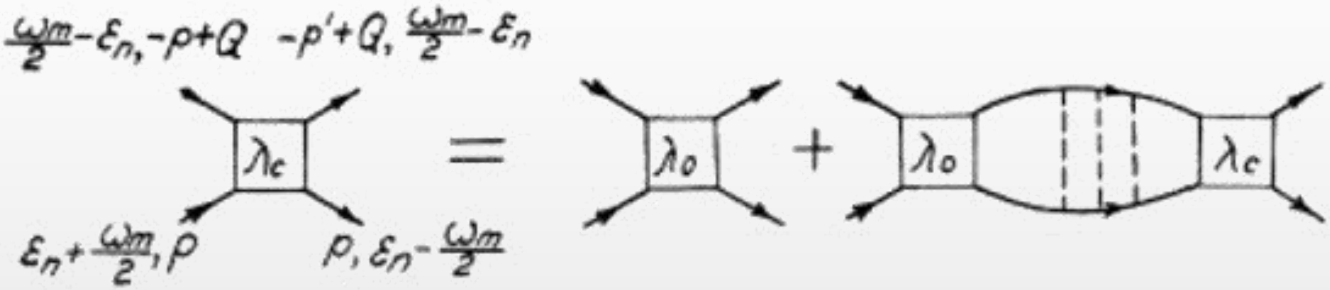
- **Diffusive Cooper ladder with MFL self-energy**



- $$\chi_{\text{pair}}^{-1} = -\frac{2N}{W} + 2\pi\nu_0 N\mathcal{S}(t), \qquad t = T/\gamma_{\text{el}}$$

Pairing in the MFL 1: Semiclassics

- **Diffusive Cooper ladder with MFL self-energy**

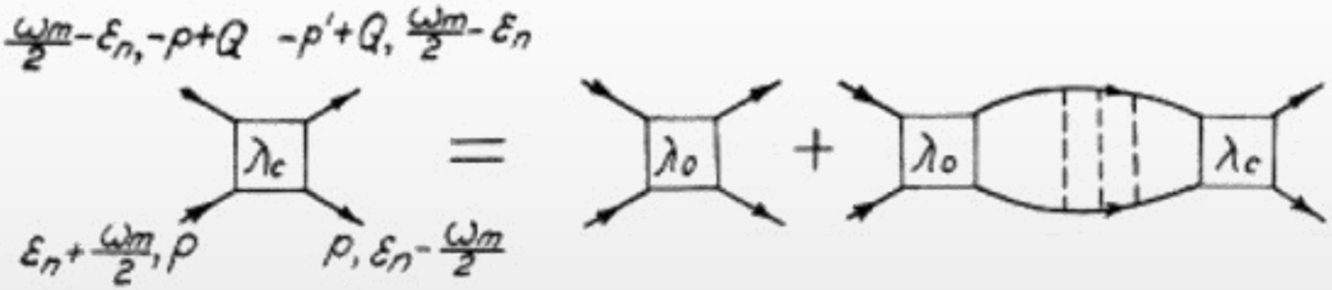


- $$\chi_{\text{pair}}^{-1} = -\frac{2N}{W} + 2\pi\nu_0 N \mathcal{S}(t), \qquad t = T/\gamma_{\text{el}}$$

- $$\mathcal{S}(t) = \int_{-1/t}^{1/t} \frac{dy}{2\pi} \frac{\tanh(y)}{\left[y\,\mathcal{A}_{\text{MFL}}(y) + i\left(4t\gamma_{\text{el}}\tau_\varphi\right)^{-1}\right]}$$

Pairing in the MFL 1: Semiclassics

- **Diffusive Cooper ladder with MFL self-energy**



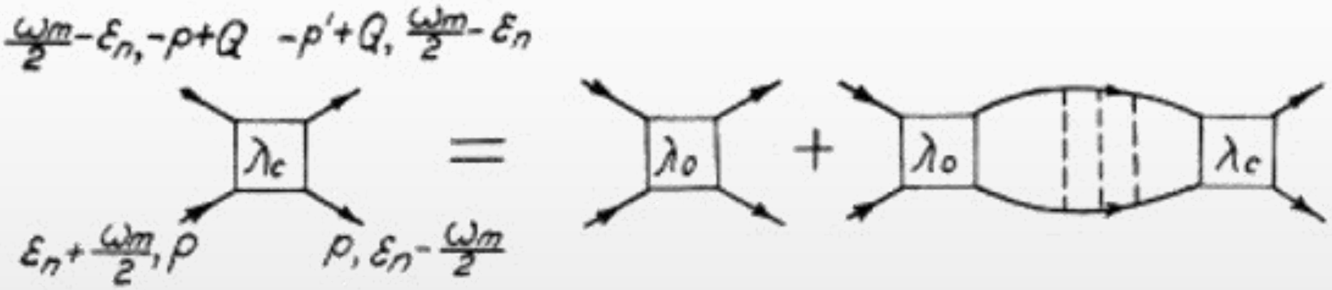
- $$\chi_{\text{pair}}^{-1} = -\frac{2N}{W} + 2\pi\nu_0 N \mathcal{S}(t), \qquad t = T/\gamma_{\text{el}}$$

- $$\mathcal{S}(t) = \int_{-1/t}^{1/t} \frac{dy}{2\pi} \frac{\tanh(y)}{\left[y\,\mathcal{A}_{\text{MFL}}(y) + i\left(4t\gamma_{\text{el}}\tau_\varphi\right)^{-1}\right]}$$

- $$\mathcal{A}_{\text{MFL}}(y) = 1 + \bar{g}^2 \ln\left[\frac{\omega_c/T}{\max(J, 2|y|)}\right]$$

Pairing in the MFL 1: Semiclassics

- **Diffusive Cooper ladder with MFL self-energy**



- **At low-*T*, can drop dephasing**
- ***Strong suppression of T_c: (Planckian dissipation: no well-defined fermion quasiparticles)***

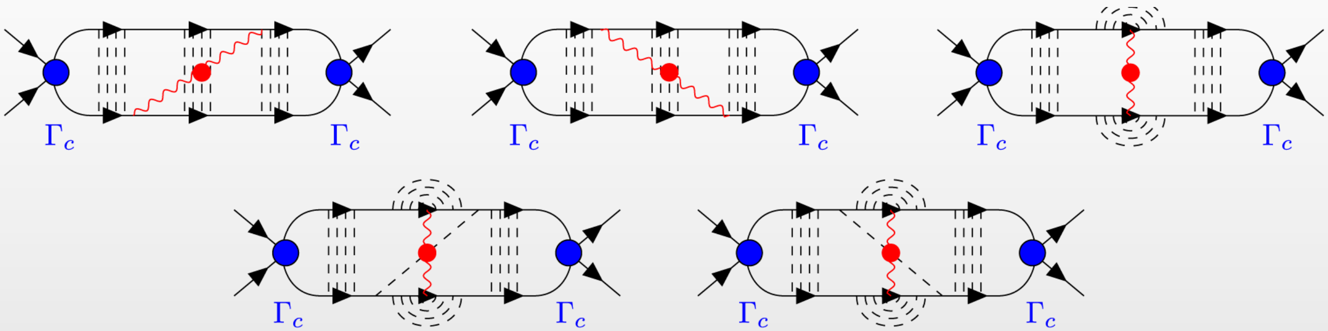
Cf. Raghu, Torroba, Wang 2015

$$T_c^{\mathcal{S}} \sim \Lambda \, \exp\left[-\frac{1}{\bar{g}^2}\left(e^{\bar{g}^2/\nu_0 W} - 1\right)\right] \ll \Lambda \, e^{-1/\nu_0 W}$$

Pairing in the MFL 2: Quantum interference

- **Interference-mediated mixing of Cooper, SU(N) boson:**

  Maekawa-Fukuyama processes



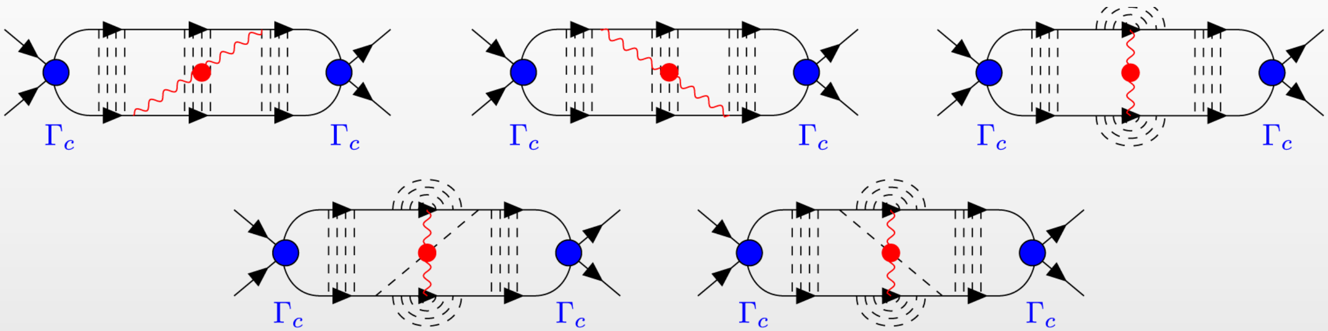
- $$\chi_{\text{pair}}^{-1} = -\frac{2N}{W} + 2\pi\nu_0 \left[N\mathcal{S}(t) + \mathcal{Q}(t)\right]$$

Related: SC near a FM QCP Nosov, Burmistrov, Raghu 2023

Pairing in the MFL 2: Quantum interference

- **Interference-mediated mixing of Cooper, SU(N) boson:**

  Maekawa-Fukuyama processes



- $$\chi_{\text{pair}}^{-1} = -\frac{2N}{W} + 2\pi\nu_0 \left[N\mathcal{S}(t) + \mathcal{Q}(t) \right]$$

- $$\mathcal{Q}(t) \simeq \begin{cases} \dfrac{\bar{g}^2\, \mathcal{C}(t)}{\pi\, \nu_0\, D\, t}, & t \gtrsim t_*, \\ 7.25\sqrt{\dfrac{N\bar{g}^2}{2^3\pi^4\, \nu_0\, D\, t}}, & t \ll t_* \end{cases}$$

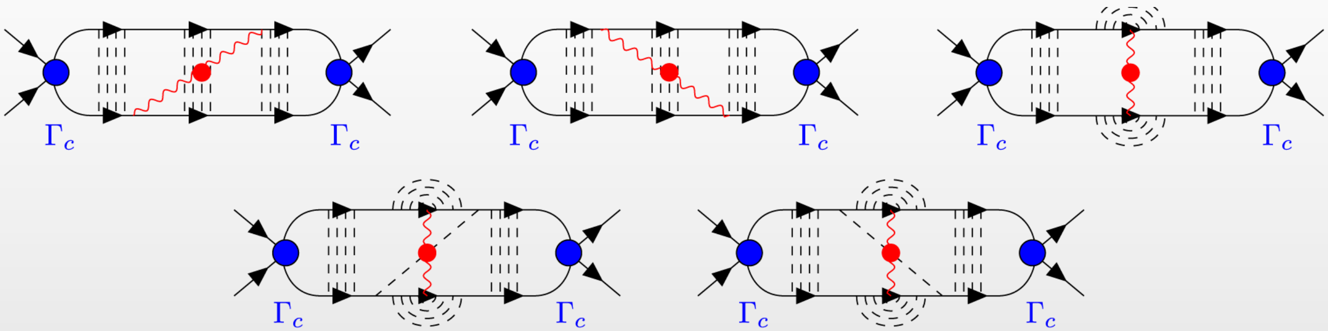
Landau damping:

$$t_* = \frac{2\pi^2\bar{g}^2\nu_0}{\alpha_m N}$$

Pairing in the MFL 2: Quantum interference

- **Interference-mediated mixing of Cooper, SU(N) boson:**

  Maekawa-Fukuyama processes



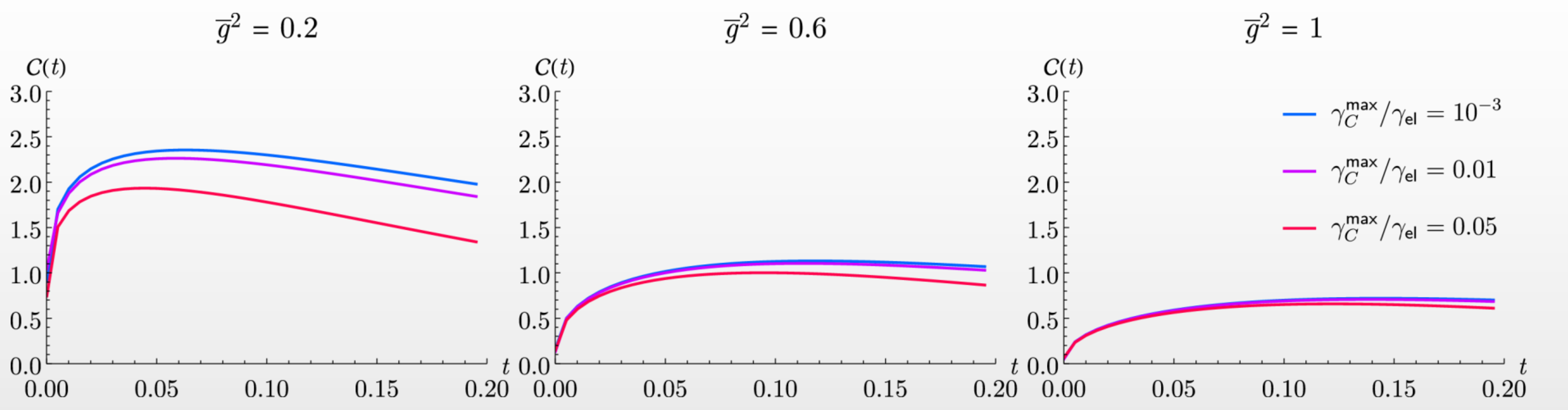
- Quantum correction is a power-law in temperature $t = T/\gamma_{el}$

- $$\mathcal{Q}(t) \simeq \begin{cases} \dfrac{\bar{g}^2\, \mathcal{C}(t)}{\pi\, \nu_0\, D\, t}, & t \gtrsim t_*, \\ 7.25\sqrt{\dfrac{N\bar{g}^2}{2^3 \pi^4\, \nu_0\, D\, t}}, & t \ll t_* \end{cases}$$

Landau damping:

$$t_* = \frac{2\pi^2 \bar{g}^2 \nu_0}{\alpha_m N}$$

Pairing in the MFL 2: Quantum interference



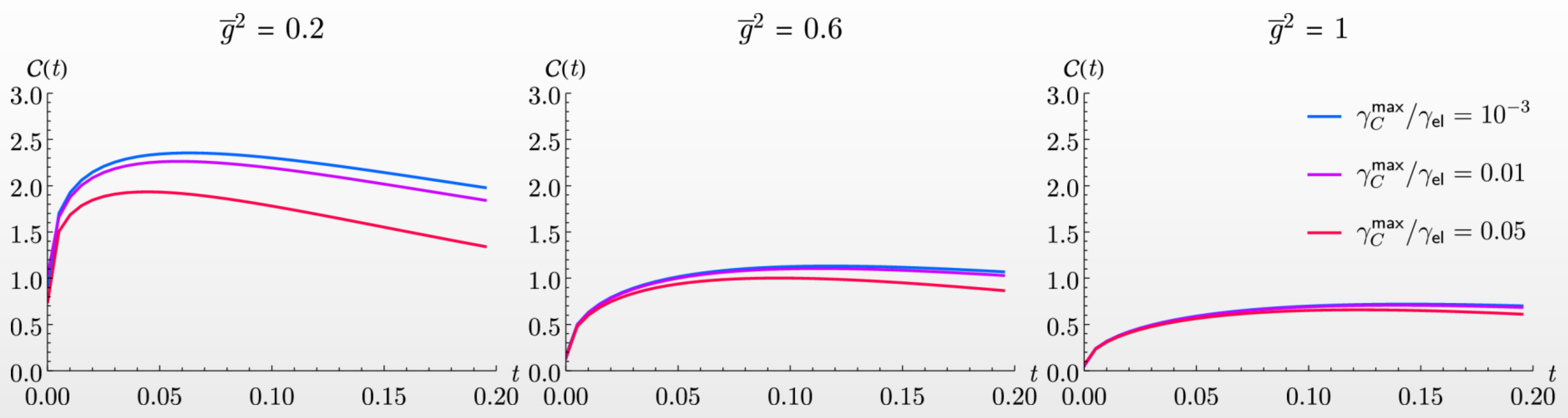
- **Quantum correction is a power-law in temperature $t = T/\gamma_{\rm el}$**

- $$\mathcal{Q}(t) \simeq \begin{cases} \dfrac{\bar{g}^2\,\mathcal{C}(t)}{\pi\,\nu_0\,D\,t}, & t \gtrsim t_*, \\ 7.25\sqrt{\dfrac{N\bar{g}^2}{2^3\pi^4\,\nu_0\,D\,t}}, & t \ll t_* \end{cases}$$

Landau damping:

$$t_* = \frac{2\pi^2\bar{g}^2\nu_0}{\alpha_m N}$$

Pairing in the MFL 2: Quantum interference



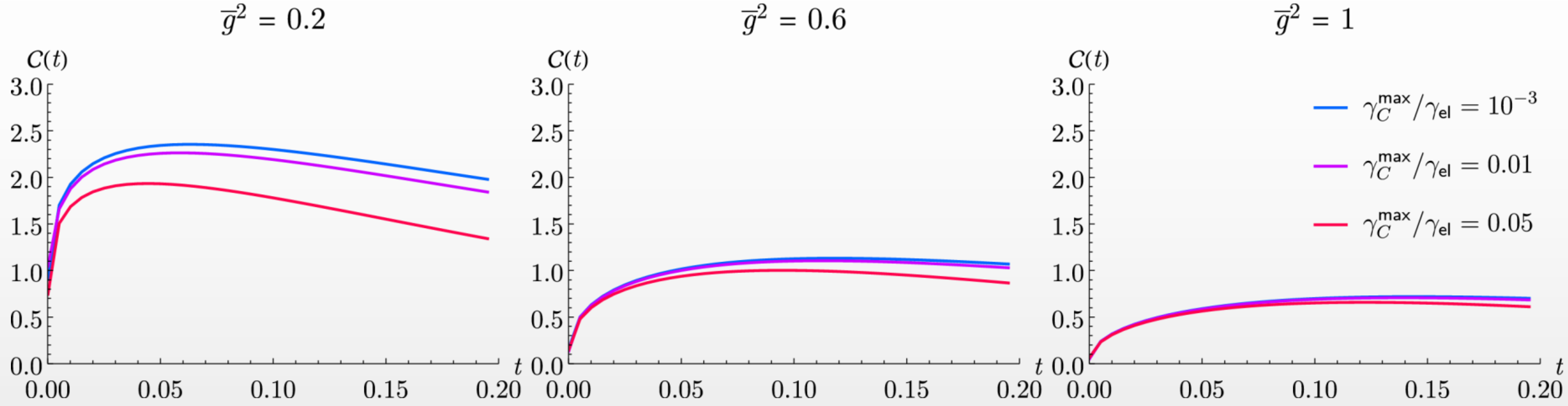
- $$\chi_{\text{pair}}^{-1} = -\frac{2N}{W} + 2\pi\nu_0 \left[N\mathcal{S}(t) + \mathcal{Q}(t) \right]$$

- $$\mathcal{Q}(t) \simeq \begin{cases} \dfrac{\bar{g}^2\, \mathcal{C}(t)}{\pi\, \nu_0\, D\, t}, & t \gtrsim t_*, \\ 7.25\sqrt{\dfrac{N\bar{g}^2}{2^3\pi^4\, \nu_0\, D\, t}}, & t \ll t_* \end{cases}$$

Landau damping:

$$t_* = \frac{2\pi^2\bar{g}^2\nu_0}{\alpha_m N}$$

Pairing in the MFL 2: Quantum interference



$$T_c \sim \begin{cases} \dfrac{g^2}{\sigma_{\text{dc}}} (\nu_0 W_{\text{eff}})\, \mathcal{C}(t_c), & T \gtrsim T_*, \\ \dfrac{g^2}{\sigma_{\text{dc}}} (\nu_0 W_{\text{eff}})^2 , & T_c \ll T_* \end{cases} \qquad T_* \sim \frac{g^2 \nu_0}{\alpha_m N}$$

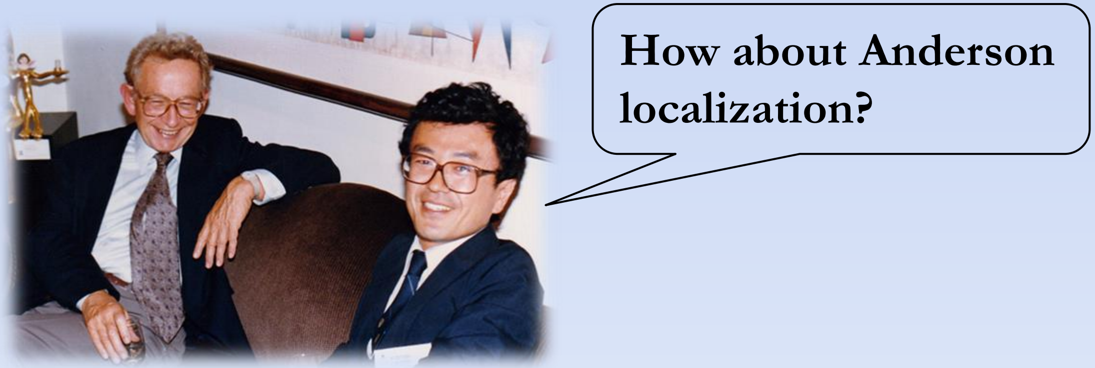
- **Semiclassical ladder ~ constant, renormalizes $W \Rightarrow W_{\text{eff}}$**
- **Power-law from quantum correction due to critical boson**
- **Interference: Larger T_c from *smaller* normal-state σ_{dc}**

Summary: Interference-mediated pairing in dirty MFLs

1. **Pairing amplitude can be enhanced by fractality (pairing of exact eigenstates picture, no long-ranged Coulomb)**

2. **T_c suppression due to interference with Coulomb: Maekawa and Fukuyama (SIT precursor)**

3. **1 and 2 are the same interference mechanism!**



Summary: Interference-mediated pairing in dirty MFLs

1. Pairing amplitude can be enhanced by fractality
   (pairing of exact eigenstates picture, no long-ranged Coulomb)

2. T_c suppressi... terference ... omb:
   Maekawa an... (SIT prec...

3. 1 and 2 are t... erference ... n!

4. **Model dirty MFL:** **N-fermions, SU(N) matrix bosons, disorder smearing**

5. **Semiclassical pairing susceptibility strongly suppressed**

6. **Quantum (Maekawa-Fukuyama) contribution is power-law in *T* due to quantum-critical bosons: interference-mediated pairing!**

- **Wu, Lee, Foster** PRB **108**, 214506 (2023)

Summary: Interference-mediated pairing in dirty MFLs

1. Pairing amplitude can be enhanced by fractality (pairing of exact eigenstates picture, no long-ranged Coulomb)
2. T_c suppression due to interference with Coulomb: Maekawa and Fukuyama (SIT precursor)
3. 1 and 2 are the same interference mechanism!
4. Model dirty MFL: N-fermions, SU(N) matrix bosons, disorder smearing
5. Semiclassical pairing susceptibility strongly suppressed
6. Quantum (Maekawa-Fukuyama) contribution is power-law in T due to quantum-critical bosons: interference-mediated pairing!
7. **Quantum interference can survive in hydrodynamic modes despite Planckian dissipation of quasiparticles…?!**