2D Majorana fluids: Quantized transport, universal statistics, and instabilities at the surface of a topological superconductor

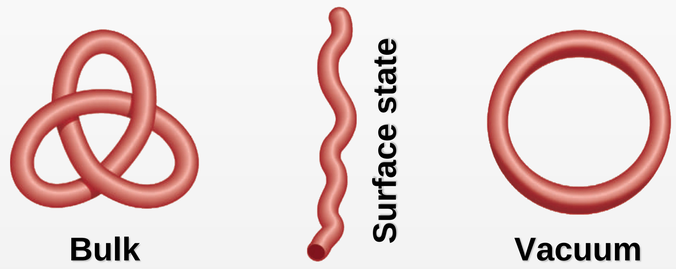
Hong-Yi Xie, Yang-Zhi Chou, and Matthew S. Foster, Rice University

Quantum Topology in Materials

Topological phases in materials physics:

Quantum holography in condensed matter

- Bulk electronic structure is insulating, "knotted"
- Knot unties at the surface: metallic surface states with anomalous properties

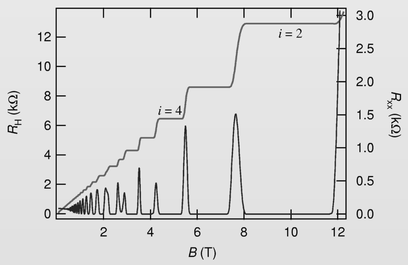


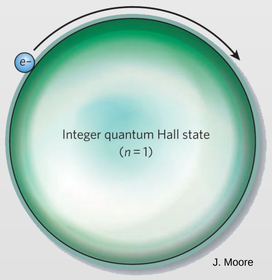
Integer quantum Hall effect

K. von Klitzing 1980, Noble Prize in Physics 1985

- Electrons in a quantum well, large magnetic field
- Anomalous "chiral" edge state
- Topologically quantized Hall resistance

$$R_H = \frac{h}{\nu e^2}, \quad \nu \in 1, 2, \ldots$$



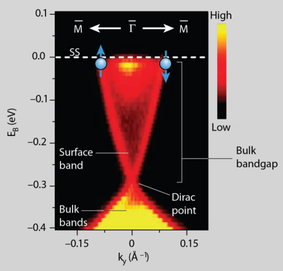


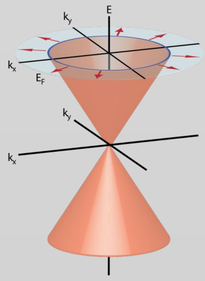
Quantum Topology in 3D: Z_2 Topological Insulators

New Topological Phases in 2D, 3D

- Topology without a magnetic field Kane and Mele 2005
- 3D Topological insulators with 2D Dirac surface states
- Surface states mapped with optics (photoemission)

Spaceballs the Movie





Y. Xia et al.; D. Hsieh et al. 2009

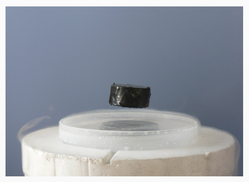
Fu, Kane, Mele; Moore and Balents 2007; Qi, Hughes, Zhang 2008; R. Roy 2009

- Drawback: Surface transport not quantized, does not probe knot winding number

Topological Superconductors: Universal Majorana Transport

Superconductivity

Collective motion of electron pairs at low temperatures



Mai-Linh Doan, Wikipedia

- Superfluidity: Electrical resistance is zero
- No heat or spin transport in the superfluid
- Topological superconductor: Theorized to possess a charge neutral surface fluid of "Majorana" fermions

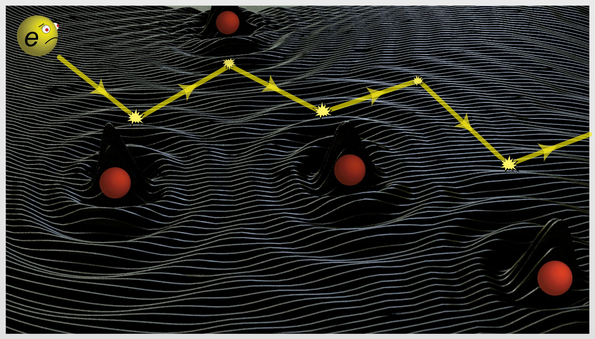
Volovik 1988; Salomaa and Volovik 1988; Read and Green 2000; Schnyder, Ryu, Furusaki, Ludwig 2008

Spin and Heat Transport in a 2D Majorana Fluid

Surface Majorana fluid can carry spin or heat

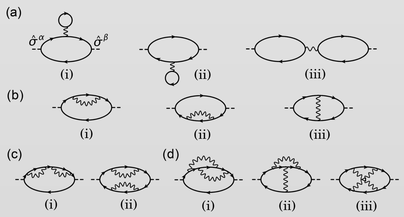
In 2D, wave interference dominates transport; quantum conductance corrections due to

- Multiple scattering off of impurities Ludwig et al. 1994; Tsvelik 1995; Ostrovsky, Gornyi, Mirlin 2006
- Scattering off of impurity-induced density ripples



Ripples by Leonardo Nuevo Arenas

Interaction (Altshuler-Aronov) corrections to second order:

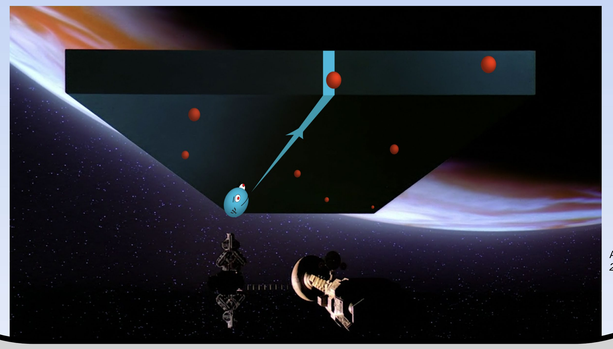


Result: All corrections vanish

Xie, Chou, Foster 2014

- Anomalous surface time-reversal symmetry: Majorana fluid remains ripple-free even with dirt
- Spin, heat conductivities encode topology:

$$\sigma_s = \frac{|\nu|}{\pi h}\left(\frac{\hbar}{2}\right)^2, \quad \kappa = \frac{|\nu|}{\pi h}\frac{\pi^2 k_B^2 T}{3\gamma}, \quad \gamma \in 1, 2$$



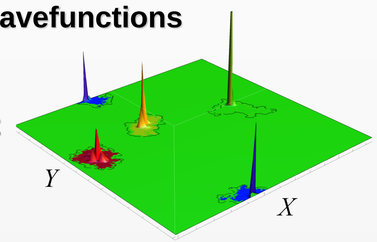
Adapted from 2010

Disorder and interactions; Delocalization and Instabilities

Universal transport; Disorder has no effect?

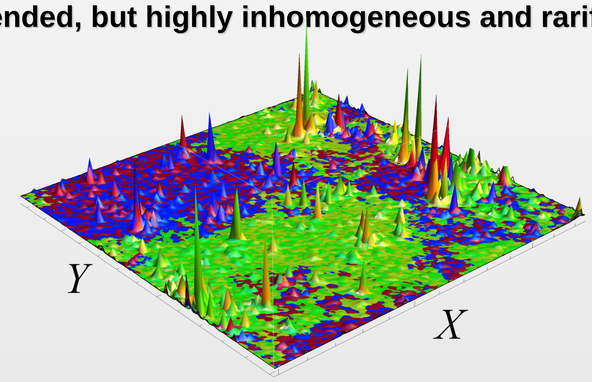
- Ordinary 2D electron gas: arbitrarily weak disorder localizes all wavefunctions
- Anderson insulator: localized states cannot carry current

Anderson 1958; Abrahams, Anderson, Licciardello, Ramakrishnan 1979



Surface Majorana states cannot be localized by disorder (topological protection) Schnyder, Ryu, Furusaki, Ludwig 2008

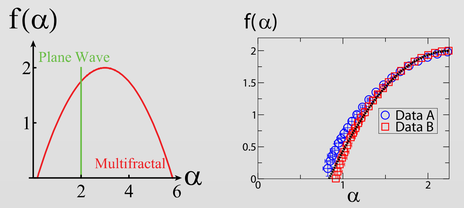
- Despite featureless density, Wavefunctions are extended, but highly inhomogeneous and rarified



- *Universal* wavefunction statistics can be obtained exactly using 2D conformal field theory

Class	Embedding	$\lvert\nu\rvert$	
CI	$\mathrm{SO}(4nk)_1 \supset \mathrm{Sp}(2n)_k \oplus \mathrm{Sp}(2k)_n$	$2k$	$(k \geq 1)$
AIII	$\mathrm{U}(nk)_1 \supset \mathrm{U}(n)_k \oplus \mathrm{SU}(k)_n$	k	$(k \geq 2)$
DIII	$\mathrm{SO}(nk)_1 \supset \mathrm{SO}(n)_k \oplus \mathrm{SO}(k)_n$	k	$(k \geq 3)$

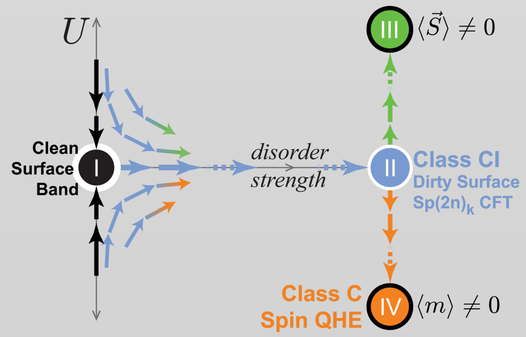
Nersesyan, Tsvelik, Wenger 1994
Caux, Kogan, Tsvelik 1996
Mudry, Chamon, Wen 1996
Foster and Yuzbashyan 2012
Foster, Xie, Chou 2014



Foster, Xie, Chou 2014
Chou and Foster 2014

- *Nesting* of different wavefunctions (overlapping probability peaks): Fluctuations enhance interactions
- Result: Not always protected. Even weak disorder and interactions can destroy some surface states

Foster and Yuzbashyan 2012; Foster, Xie, Chou 2014



1) M.S.F. and E. Yuzbashyan, Phys. Rev. Lett. (2012)
2) M.S.F., H.-Y. X., and Y.-Z. C., Phys. Rev. B (2014)
3) Y.-Z. C. and M.S.F., Phys. Rev. B (2014)
4) H.-Y. X, Y.-Z. C., and M.S.F., arXiv:1405.7730