Helical Quantum Edge Gears in 2D Topological Insulators

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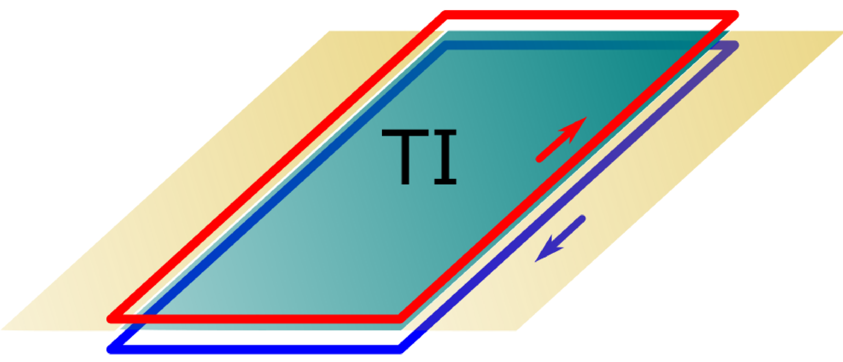
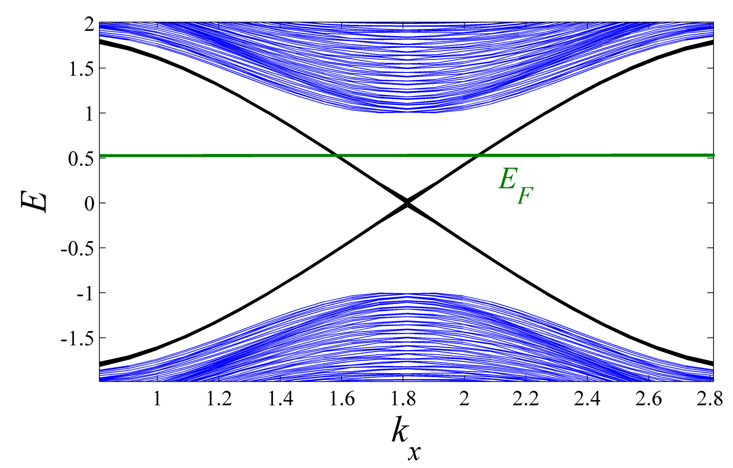
2D Topological Insulators and Helical Edge States



2D Topological Insulators:
- Kane-Mele, BHZ models
- HgTe, InAs/GaSb, ...
- Time reversal symmetry
- Robust edge states

Edge States:
- Kramers pair
- Ballistic conduction at $T=0$
- Luttinger interactions
- Protection from localization



Helical Luttinger Liquid and Transport at $T=0$

Edge Theory: 1D Dirac Fermion:

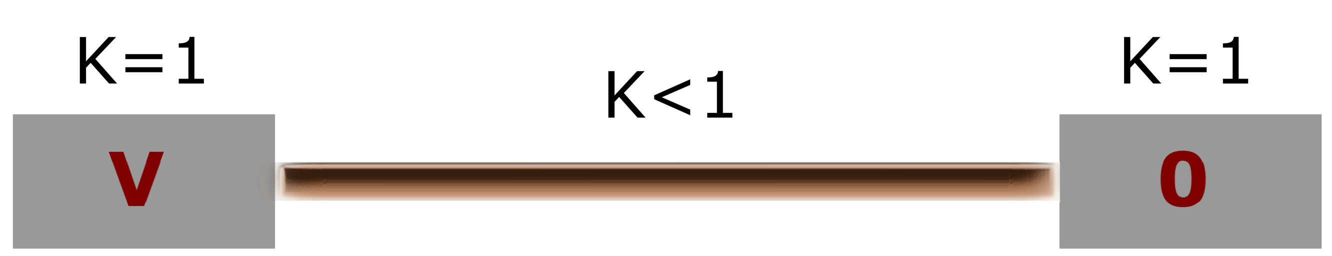
$$\hat{H}_0 = -i\hbar v_F \int dx \left[R^\dagger(x)\partial_x R(x) - L^\dagger(x)\partial_x L(x)\right] + \hat{H}_{LL}.$$

Bosonization: $n = \frac{1}{\sqrt{\pi}}\partial_x\theta$, $I = -\frac{1}{\sqrt{\pi}}\partial_t\theta$,

$$\hat{H}_0 \to \hat{H}_{b,0} = \frac{\hbar v}{2}\int dx \left[K(\partial_x\phi)^2 + \frac{1}{K}(\partial_x\theta)^2\right].$$

$K<1$: Repulsive
$K>1$: Attractive

Transport in 1D:



Boson propagator obeys

$$\frac{\omega^2}{v(x)K(x)}\tilde{G}(\omega;x,x') + \partial_x\left[\frac{v(x)}{K(x)}\partial_x\tilde{G}(\omega;x,x')\right] = \delta(x-x').$$

The current can be computed via

$$\langle I_1(x)\rangle = i\frac{e^2}{\pi\hbar}\frac{V}{L}\int_{-L/2}^{L/2} dx' \lim_{\omega\to 0}\left[\omega\,\tilde{G}^{(R)}(\omega;x,x')\right] = \frac{e^2}{h}V$$

Maslov and Stone 1995; Ponomarenko 1995; Safi and Schulz 1995.

DC conductance is independent of K!

Rashba Spin Orbit Coupling and Backscattering Terms

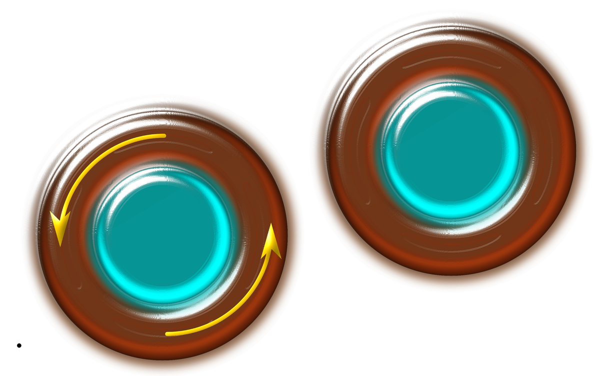
- No spin conservation
- "Spin-flip" backscatterings

Intraedge Interactions:

Leading irrelevant term is

$$\hat{H}_W = W\int dx : \left[e^{i2k_Fx}L^\dagger RR^\dagger(-i\partial_x R) + e^{-i2k_Fx}R^\dagger LL^\dagger(-i\partial_x L) + \text{H.c.}\right]: .$$

S_z conserving: No drag



Interedge Interactions:

Two-Particle Backscattering

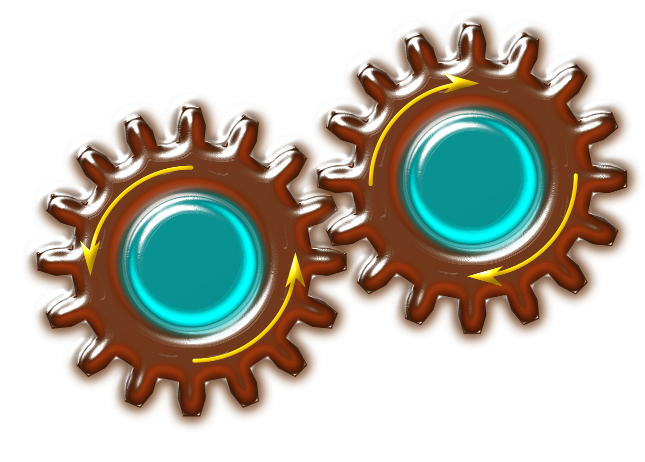
$$\hat{H}_- = U_-\int dx\left[e^{i2(k_{F1}-k_{F2})x}L_1^\dagger R_1 R_2^\dagger L_2 + \text{H.c.}\right],$$

$$\hat{H}_+ = U_+\int dx\left[e^{i2(k_{F1}+k_{F2})x}L_1^\dagger R_1 L_2^\dagger R_2 + \text{H.c.}\right].$$

One-Particle Backscattering

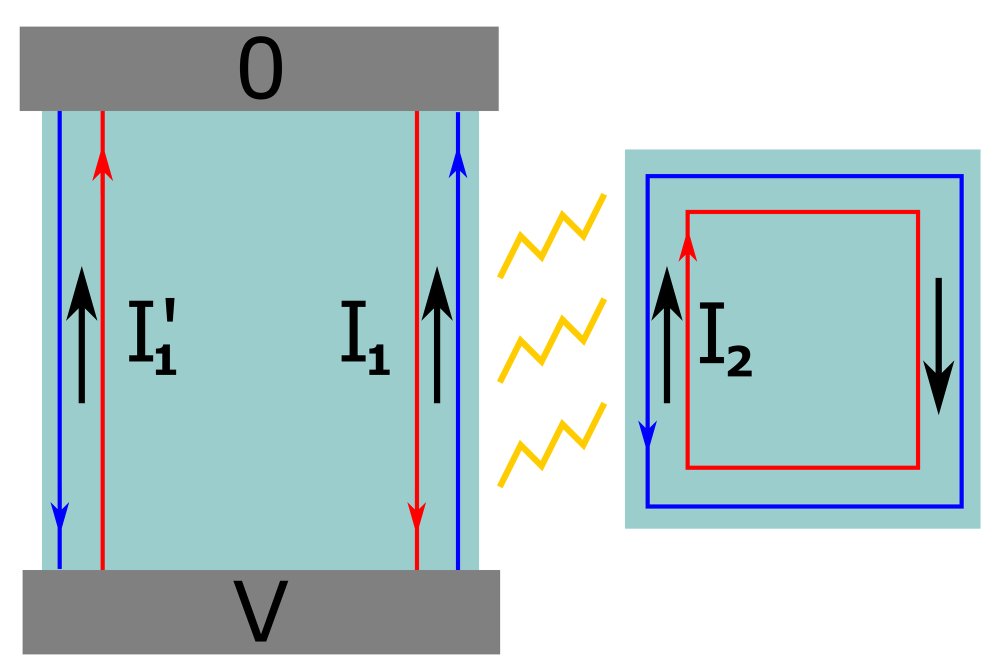
$$\hat{H}_U = \sum_{a=1,2} U\int dx\left[e^{-i2k_{Fa}x}R_a^\dagger L_a R_{\bar{a}}^\dagger R_{\bar{a}} - e^{i2k_{Fa}x}L_a^\dagger R_a L_{\bar{a}}^\dagger L_{\bar{a}} + \text{H.c.}\right].$$

Rashba: "Locking" drag



Correlated Transport in the Low-T Locking Regime

Assuming parameters (K, k_F, ...) in each edge are the same.



Transport in two-edge setup:

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix}\begin{bmatrix} V_1 = V \\ V_2 = 0 \end{bmatrix}, \quad G_{ij} = \sigma_{ij}/L.$$

At Low temperatures,
- Immune to disorder
- Perfect drag, $I_1 = I_2$, $G_1 = \frac{1}{1+1/K}\frac{e^2}{h}$
- Edge with I_1' contributes $G_1' = \frac{e^2}{h}$

Conditions:
$L \gg \hbar v/\Delta$ and $k_BT \ll \Delta$.
Δ: Gap from $\hat{H}_-$

DC conductance: $G = \frac{2K+1}{K+1}\frac{e^2}{h}$, K-dependent

Dissipative Regime (Above Locking T)

Overview:
- Backscattering interactions are treated as perturbations
- Forwardscattering disorder is included
- Backwardscattering disorder is not considered
- Standard effective potential method $\to$ Boson propagator

Kubo Formula:

$$\sigma_{11} = -\frac{1}{\pi}\frac{e^2}{\hbar}\lim_{\omega\to 0}\text{Im}\left[\omega\,\mathcal{G}_{11}^{(R)}(\omega,k)\right] = \frac{e^2}{2h}\left[\frac{1}{\Xi_+ + \Xi_U + \Xi_W} + \frac{1}{\Xi_- + \Xi_U + \Xi_W}\right].$$

Ξ_W, $\Xi_\pm$, and Ξ_U correspond to $\hat{H}_W$, $\hat{H}_\pm$, and $\hat{H}_U$. Ξ_W is alway sub-leading.

Summary of finite temperature:

For clean edges and $k_BT \gg \hbar v k_F$,

$$\sigma_{11} \sim \begin{cases} T^{-4K+3}, & \text{for } K \leq 1, \\ T^{-2K+1}, & \text{for } K > 1. \end{cases}$$

With smooth disorder and $k_BT \ll g_\eta/\hbar v$ (g_η: disorder strength),

$$\sigma_{11} \sim \begin{cases} T^{-4K+2}, & \text{for } K \leq 1, \\ T^{-2K}, & \text{for } K > 1. \end{cases}$$

Single Edge: Clean Conductivity

Boson propagator:

$$\langle\theta\,\theta\rangle^{(R)}(\omega,k) = \left[\frac{1}{vK}\left(\omega^2 - v^2k^2\right) - \Pi_W^{(R)}(\omega,k)\right]^{-1},$$

where $\text{Im}\left[\Pi_W^{(R)}(\omega,k)\right] = -2\omega\Xi_W + \boldsymbol{O}\left(\omega^2\right)$,

Clean Conductivity:

$$\sigma_{dc} = \frac{e^2}{h}\frac{1}{\Xi_W} = \frac{e^2}{h}\frac{(\hbar v)^2 l_T^3}{W^2\pi^3}\frac{6\left[\cosh(k_F l_T) + 1\right]}{\left[\left(\frac{k_F l_T}{2\pi}\right)^4 + \frac{5}{2}\left(\frac{k_F l_T}{2\pi}\right)^2 + \frac{9}{16}\right]} \sim \begin{cases} T^{-3}, & \text{for } k_BT \gg \hbar v k_F, \\ T\,e^{\frac{\hbar v k_F}{k_BT}}, & \text{for } k_BT \ll \hbar v k_F, \end{cases}$$

where $l_T \equiv \hbar v/k_BT$ denotes the thermal de Broglie wavelength.

Outlook

- Negative drag ($\hat{H}_+$ dominates) still works
- Nonequlibrium spectroscopy
- Noise measurement
- Similar setup for other topological materials