# Distribution functions and probes of far-from-equilibrium topological matter

Yunxiang Liao[1,2], Matthew S. Foster[2]

[1]Applied Physics Program, Rice University, Houston, Texas 77005 USA,
[2]Department of Physics and Astronomy, Rice University, Houston, Texas 77005 USA

## Introduction

**What is a quantum quench?**
Creating an out of equilibrium state of an isolated many body system by tuning Hamiltonian parameters

**Why study quantum quenches?**

- New dynamical phases
- Study the evolution of many body systems
- Experimental tool to study ultracold atomic systems
- An alternative approach to create p-wave superfluids

## Background

**2D p+ip superconductor**

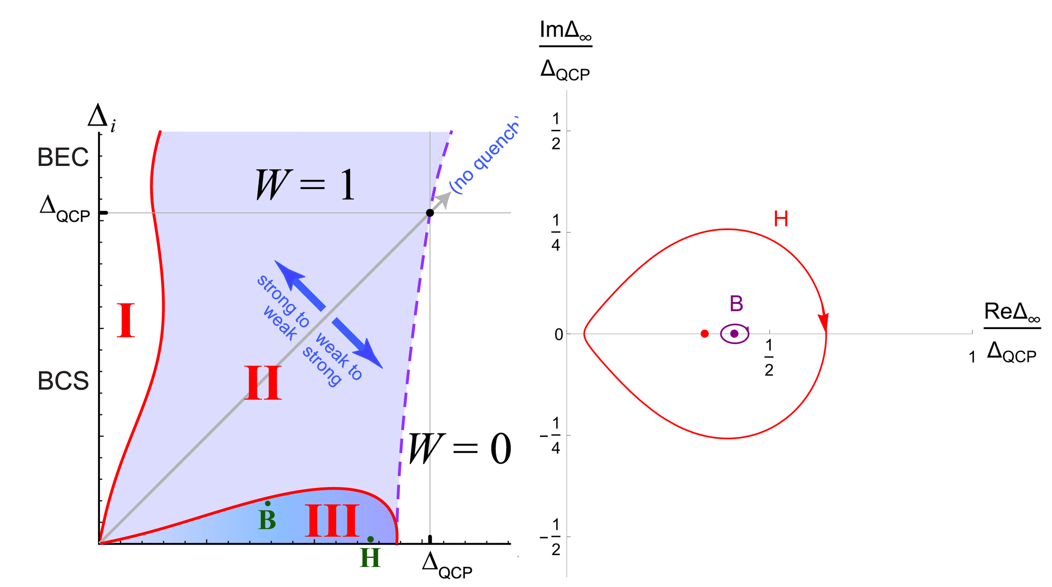
$$H = \sum_{\mathbf{k}} \frac{k^2}{2m} c^\dagger_{\mathbf{k}} c_{\mathbf{k}} - \frac{2G}{m} \sum_{\mathbf{k},\mathbf{q}}{}' c^\dagger_{\mathbf{k}} c^\dagger_{-\mathbf{k}} c_{-\mathbf{q}} c_{\mathbf{q}} \qquad (1)$$

- It could be realized experimentally in a gas of fermionic $^{40}K$, $^{6}Li$ atoms
- It is a topological superfluid
  - $\Delta < \Delta_{QCP}$ BCS phase, topologically non trivial
  - $\Delta > \Delta_{QCP}$ BEC phase, topologically trivial

**Quench setup**

- t<0
  before the quench, prepare the system in ground state of initial Hamiltonian $H_i\left\{G_i, \Delta_0^{(i)}\right\}$
- t=0
  suddenly quench the coupling strength $G_i \to G_f$
- t>0
  after the quench, the system evolves as a superposition of eigenstates of post-quench Hamiltonian $H_f\left\{G_f, \Delta_0^{(f)}\right\}$

**Quench phase diagram** [1, 2]
Phase I: $\Delta(t) \to 0$
Phase II: $\Delta(t) \to \Delta_\infty e^{2i\mu_\infty t}$
Phase III: $\Delta(t) \to \Delta_\infty(t) e^{2i\mu_\infty t}$, $\Delta_\infty(t+T) = \Delta_\infty(t)$




## Method

**Radio-Frequency Spectorscopy**

- RF radiation induces transitions between the paired levels ($c_{\mathbf{k}}$) and unpaired levels ($d_{\mathbf{k}}$)

$$H_T = \mathcal{T} \sum_{\mathbf{k}} \left[ e^{i\omega_L t} c^\dagger_{\mathbf{k}} d_{\mathbf{k}} + e^{-i\omega_L t} d^\dagger_{\mathbf{k}} c_{\mathbf{k}} \right], \qquad (2)$$

- Observable: $I(\omega) \equiv \langle \frac{dN_d}{dt} \rangle$

**Tunneling**

- Tunneling between the system ($c_{\mathbf{k}}$) and a norm metal tip ($d_{\mathbf{k}}$)

$$H_T = \mathcal{T} \left[ c^\dagger(\mathbf{r}_0) d(\mathbf{r}_0) + d^\dagger(\mathbf{r}_0) c(\mathbf{r}_0) \right] \qquad (3)$$

- Observable: $I(V) \equiv \langle \frac{dN_d(\mathbf{r}_0)}{dt} \rangle$

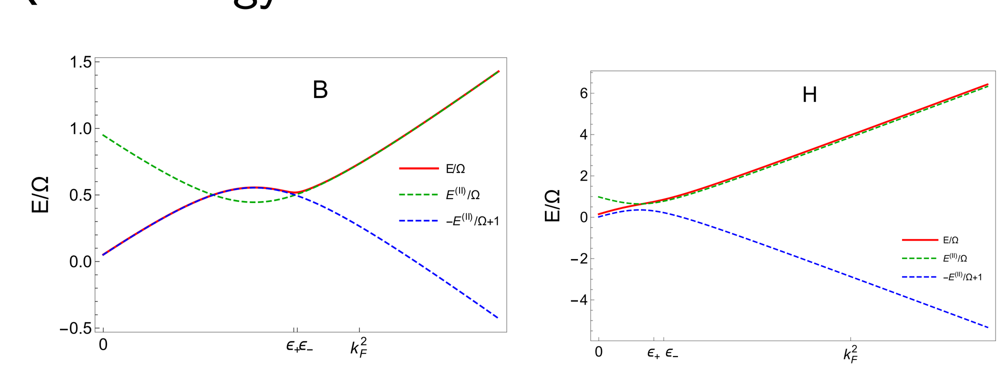
## Quench-induced Floquet phase III: Explicit solution

**Solution to BdG equation: superposition of two orthogonal Floquet states**

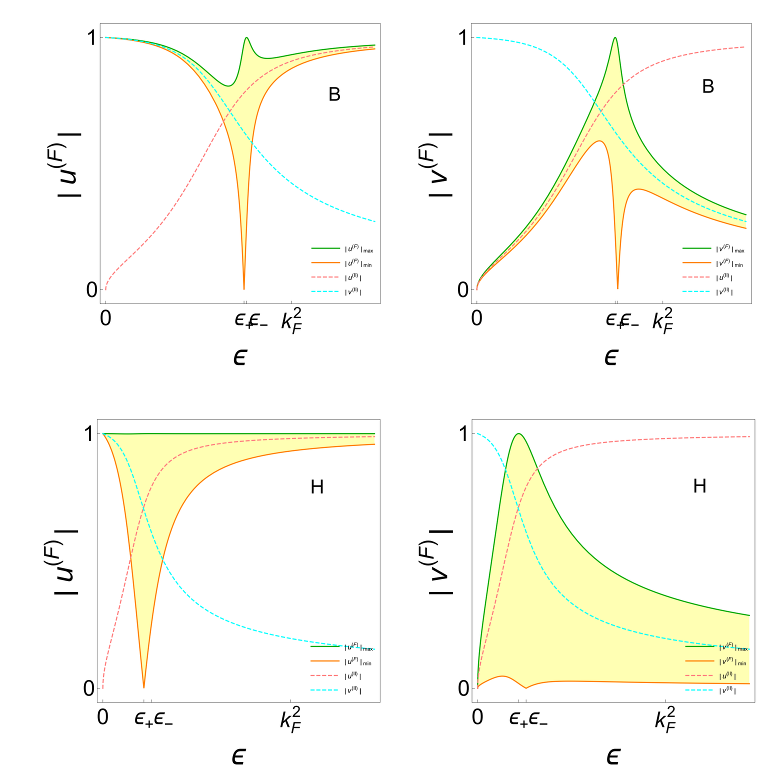
$$\begin{bmatrix} u_{\mathbf{k}}(t) \\ v_{\mathbf{k}}(t) \end{bmatrix} = \sqrt{\frac{1-\gamma_{\mathbf{k}}}{2}} \begin{bmatrix} u^{(F)}_{\mathbf{k}}(t) \\ v^{(F)}_{\mathbf{k}}(t) \end{bmatrix} e^{+iE^{(F)}_{\mathbf{k}} t} + \sqrt{\frac{1+\gamma_{\mathbf{k}}}{2}} \begin{bmatrix} v^{*(F)}_{\mathbf{k}}(t) \\ -u^{*(F)}_{\mathbf{k}}(t) \end{bmatrix} e^{-iE^{(F)}_{\mathbf{k}} t + i\Gamma_{\mathbf{k}}} \qquad (4)$$

**Floquet state**

- Quasi-energy in the extended zone scheme



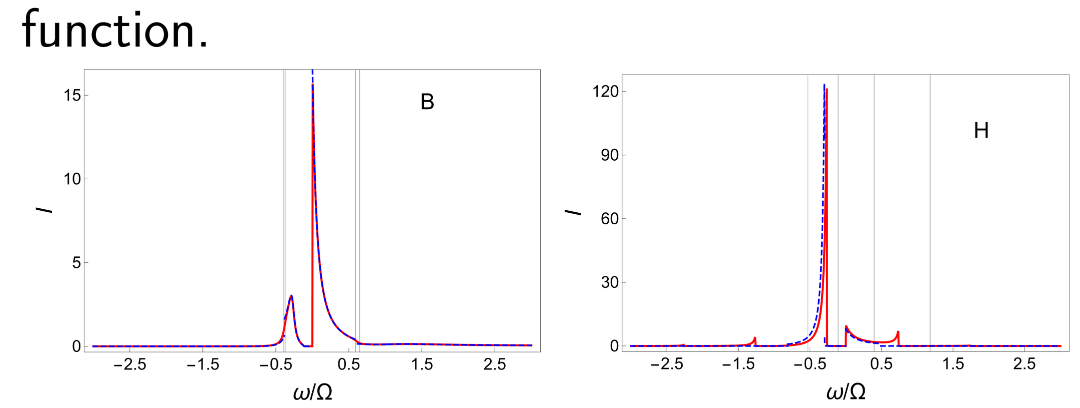

- Coherence factors



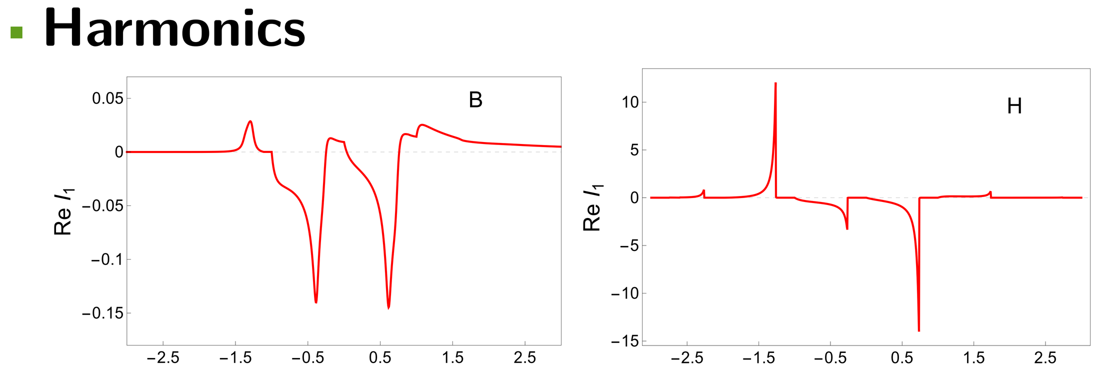

The spectrum is dramatically different from that of the post-quench states. This shows the important role the distribution function plays in the computation of the RF signal from a Floquet system. In fact, the actual spectrum of post-quench states conveys some topological information - the distribution functions exhibits an odd number of zeroes as required from the conservation of the pseudospin winding number.

## RF spectrum in the Floquet phase

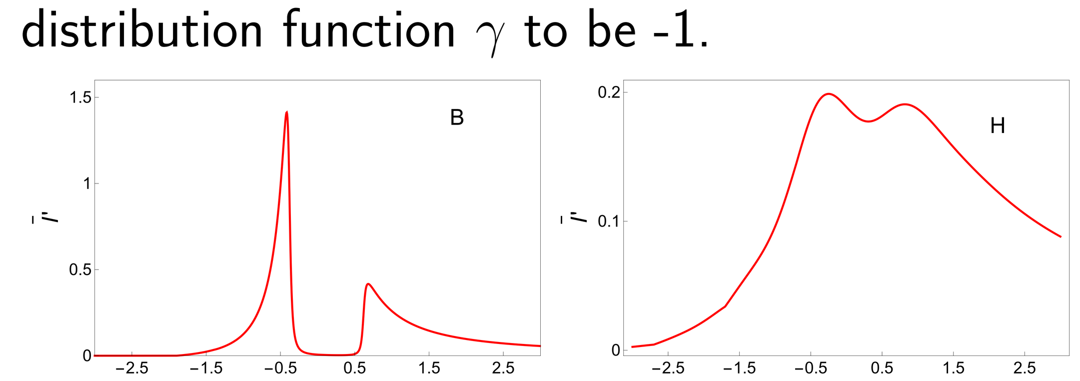
- **Time averaged current**
  The spectrum (red curve) almost overlaps with the Phase II approximation(blue dashed line) primarily because of the non-equilibrium distribution function.




- **Harmonics**



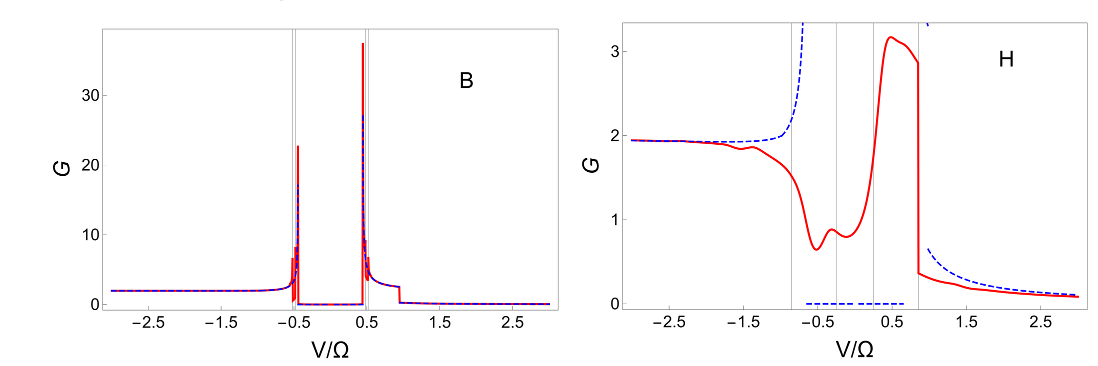

- **RF for lower Floquet Band**
  We compute the time-averaged RF current obtained from a system occupying the lower Floquet band. In another word, we set the pair distribution function $\gamma$ to be -1.



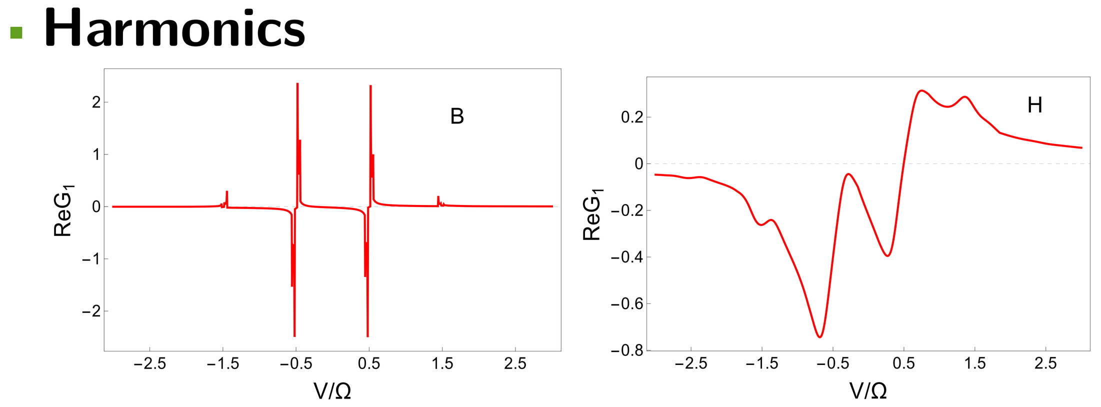

## Tunneling spectrum in the Floquet phase

While the distribution function strongly affects the RF signal, it disappears from the tunneling spectrum.

- **Time averaged differential conductance**
  The time averaged tunneling spectrum(red curve) is compared with its phase II approximation(blue dashed line) here.




- **Harmonics**




## References

[1] Matthew S Foster, Victor Gurarie, Maxim Dzero, and Emil A Yuzbashyan.
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[2] Matthew S Foster, Maxim Dzero, Victor Gurarie, and Emil A Yuzbashyan.
Quantum quench in a p+ i p superfluid: Winding numbers and topological states far from equilibrium.
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