Universal Surface Transport Coefficients of 3D Topological Superconductors†

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I. Introduction

- 3D topological superconductors (TSCs) host surface Majorana-fermion zero modes protected by time-reversal ($\mathcal{T}$) symmetry:

Class	Spin symmetry	Winding number (color) ν
CI	SU(2)	$2\mathbb{Z}$
AIII	U(1)	$\mathbb{Z}$
DIII	No	$\mathbb{Z}$

- Neglecting interactions, the surface longitudinal transport coefficients are independent of disorder at low temperature:

Spin conductance (only for CI and AIII):

$$\sigma^{s}_{xx} = \frac{|\nu|}{\pi h}\left(\frac{\hbar}{2}\right)^2$$

Thermal conductance

$$\kappa_{xx} = \frac{|\nu|}{\pi h}\frac{\pi^2 \kappa_B^2 T}{3\gamma}, \quad \gamma = \begin{cases} 1, & \text{CI and AIII} \\ 2, & \text{DIII} \end{cases}$$

- On top of disorder, interactions usually induce Altshuler-Aronov corrections to conductance, due to the scattering of electrons off of self-consistent potential induced by density fluctuations near impurities.

Q: What are the Altshuler-Aronov corrections to σ^{s}_{xx} and κ_{xx} of the TSC surface states?

II. Majorana surface theory

- Non-interacting Majorana surface Hamiltonian

$$H^{(0)} = \int d^2\mathbf{r}\, \eta^\dagger(\mathbf{r})\, \hat{h}\, \eta(\mathbf{r}), \quad \hat{h} = \hat{\boldsymbol{\sigma}}\cdot\left[-i\boldsymbol{\nabla} + \mathbf{A}_j(\mathbf{r})\,\hat{\mathfrak{t}}^j + \boldsymbol{\mathcal{A}}(\mathbf{r})\right]$$

1. Fermion field $\eta \to \eta_{\sigma,\kappa}$ with pseudospin $\sigma \in \{1,2\}$ and color $\kappa \in \{1,\cdots,|\nu|\}$; $\hat{\boldsymbol{\sigma}} = (\hat{\sigma}^1, \hat{\sigma}^2)$ and $\{\hat{\mathfrak{t}}^j\}$ being the color space symmetry generators .
2. $\mathcal{T}$ symmetry $\iff$ Chiral condition $-\hat{\sigma}^3\,\hat{h}\,\hat{\sigma}^3 = \hat{h}$.
3. $\mathcal{T}$ invariant disorder appears as vector potentials $\mathbf{A}_j(\mathbf{r})$ (intercolor scattering) and $\boldsymbol{\mathcal{A}}(\mathbf{r})$ [for AIII only, coupling to U(1) spin current].
4. Important bilinear oparators (blue and green being $\mathcal{T}$ even and odd, resp.)

Z-spin density (CI, AIII)	U(1) spin current (CI, AIII)	Pairing amplitude
$S^z(\mathbf{r}) = \eta^\dagger\,\eta(\mathbf{r})/2$	$\mathbf{J}^z(\mathbf{r}) = \eta^\dagger\,\hat{\boldsymbol{\sigma}}\,\eta(\mathbf{r})/2$	$m(\mathbf{r}) = \eta^\dagger\,\hat{\sigma}^3\,\eta(\mathbf{r})$

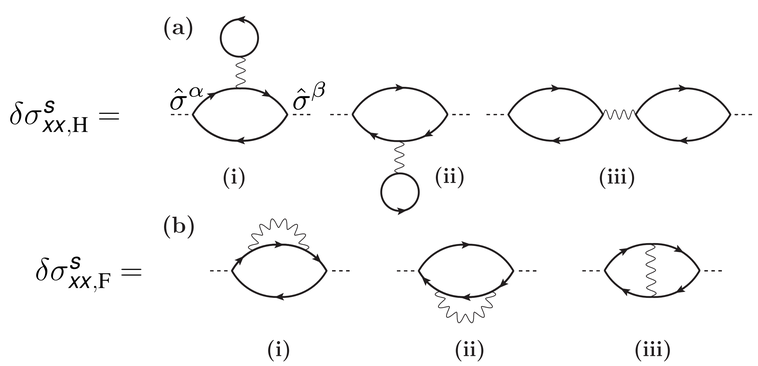
- $\mathcal{T}$ symmetric interactions

$$H^{(I)}_{\text{CI}} = \int d^2\mathbf{r}\,\left[\Gamma_s\,\mathbf{S}(\mathbf{r})\cdot\mathbf{S}(\mathbf{r}) + \Gamma_c\, m(\mathbf{r})\, m(\mathbf{r})\right],$$

$$H^{(I)}_{\text{AIII}} = \int d^2\mathbf{r}\,\left[\Gamma_s\, S^z(\mathbf{r})\, S^z(\mathbf{r}) + \Gamma_c\, m(\mathbf{r})\, m(\mathbf{r})\right],$$

$$H^{(I)}_{\text{DIII}} = \int d^2\mathbf{r}\,\Gamma_c\, m(\mathbf{r})\, m(\mathbf{r}),$$

III. Hartree-Fock spin conductivity



- Perturbative in interactions [$\sim \mathcal{O}(\Gamma_{s,c})$]

$\delta\sigma^{s}_{xx,\text{H}} = \delta\sigma^{s}_{xx,\text{F}} = 0$ in every disorder realization!

IV. Large winding number expansion

- WZNW Finkel'tein nonlinear sigma model

Class	Target manifold G	WZNW level K
CI	Sp($2nN$)	$\lvert\nu\rvert/2$
AIII	U(nN)	$\lvert\nu\rvert$
DIII	O(nN)	$\lvert\nu\rvert$

$$S[\hat{Q}] = S_0[\hat{Q}] + S_I[\hat{Q}] + \Gamma_K[\hat{Q}], \quad \hat{Q} \in G.$$

1. $S_0[\hat{Q}]$ represents the standard dynamical sigma model on the target manifold G,

$$S_0[\hat{Q}] = \frac{1}{2\lambda}\int_{\mathbf{r}} \text{Tr}\left[\nabla\hat{Q}^\dagger(\mathbf{r})\cdot\nabla\hat{Q}(\mathbf{r})\right] - h\int_{\mathbf{r}}\text{Tr}\left(|\hat{\omega}|\otimes\hat{\Sigma}^3\left[i\hat{Q}(\mathbf{r}) - i\hat{Q}^\dagger(\mathbf{r})\right]\right)$$
$$-\frac{\lambda_A}{2\lambda^2}\int_{\mathbf{r}}\text{Tr}\left[\hat{Q}^\dagger(\mathbf{r})\nabla\hat{Q}(\mathbf{r})\right]\cdot\text{Tr}\left[\hat{Q}^\dagger(\mathbf{r})\nabla\hat{Q}(\mathbf{r})\right], \quad \text{("Gade" term for AIII)}$$

λ: spin (CI and AIII) or thermal resistance (DIII)

2. $S_I[\hat{Q}]$ encodes interactions $\Gamma_{s,c}$. $\Gamma_K[\hat{Q}]$ is the WZNW action at level K.

- One-loop RG equations for λ and λ_A [$\sim \mathcal{O}(1/K)$] ($\gamma_{s,c} = 4\Gamma_{s,c}/\pi h$)

$$\text{CI:} \quad d\lambda/d\ell = \lambda^2\left[1-(K\lambda)^2\right]\left[1+\mathcal{J}(\gamma_s,\gamma_c)\right]$$
$$\text{AIII:} \quad d\lambda/d\ell = \lambda^2\left[1-(K\lambda)^2\right]\mathcal{I}(\gamma_s,\gamma_c)$$
$$\text{DIII:} \quad d\lambda/d\ell = -\lambda^2\left[1-(K\lambda)^2\right](2+\gamma_c)$$

$\mathcal{I}(\gamma_s,\gamma_c), \mathcal{J}(\gamma_s,\gamma_c)$: Altshuler-Aronov corrections.

Altshuler-Aronov corrections vanish at $\lambda = 1/K$!

V. Conclusion

- Spin and thermal conductances of TSCs surfaces are unmodified by interactions in two limits— Universal surface transport coefficients (analogous to 2D QHE).
- A simple picture: Oscillations in color and spin are forbidden by $\mathcal{T}$ symmetry.