Universal Surface Transport Coefficients of 3D Topological Superconductors

Hong-Yi Xie, Yang-Zhi Chou, and Matthew S. Foster

Physics & Astronomy Department, Rice University, Houston, USA


I. Introduction

\begin{itemize}
  \item 3D topological superconductors (TSCs) host surface Majorana-fermion zero modes protected by time-reversal (T) symmetry:
  \end{itemize}

<table>
<thead>
<tr>
<th>Class</th>
<th>Spin symmetry</th>
<th>Winding number (color) ν</th>
</tr>
</thead>
<tbody>
<tr>
<td>CI</td>
<td>SU(2)</td>
<td>Z</td>
</tr>
<tr>
<td>AllI</td>
<td>U(1)</td>
<td>Z</td>
</tr>
<tr>
<td>AllII</td>
<td>U(1)</td>
<td>Z</td>
</tr>
</tbody>
</table>

Neglecting interactions, the surface longitudinal transport coefficients are independent of disorder at low temperature:

Spin conductance (for CI and AllI):
\[ \sigma_{xx} = \frac{|ν|}{\pi h} \left( \frac{2}{2} \right)^2 \]

Thermal conductance
\[ \kappa_{xx} = \frac{|ν|}{\pi h} \frac{\pi^2 k_B T}{3\gamma}, \quad \gamma = \begin{cases} 1, & \text{CI and AllI} \\ 2, & \text{AllII} \end{cases} \]

On top of disorder, interactions usually induce Altshuler-Aronov corrections to conductance, due to the scattering of electrons off of self-consistent potential induced by density fluctuations near impurities.

Q: What are the Altshuler-Aronov corrections to \( \sigma_{xx} \) and \( \kappa_{xx} \) of the TSC surface states?

II. Majorana surface theory

Non-interacting Majorana surface Hamiltonian
\[ H^{(0)} = \int d^2 r \left[ \eta(r) \hat{h} \eta(r) \right], \quad \hat{h} = \hat{\sigma} \cdot \left[ -i \nabla + A_1(r) \right] \hat{v} + A(r) \]

\( \hat{\sigma} \) is a pseudospin, \( \eta \) is a Majorana-fermion zero mode (color), and \( \hat{v} \) is the color space symmetry generator.

\( T \) symmetry implies Chiral condition \( -\delta^2 \hat{h} \delta^2 = \hat{h} \).

T invariant disorder appears as vector potentials \( \Delta \) (intercolor scattering) and \( A(r) \) (for AllI only, coupling to U(1) spin current).

Important bilinear operators (blue and green being \( T \) even and odd, resp.):
\[ \begin{align*}
  \delta \sigma_{xx}^{\text{H}} & = \frac{1}{2} \int d^2 r \left[ \Gamma_1 S(r) \cdot S(r) + \Gamma_c m(r) m(r) \right], \\
  \delta \sigma_{xx}^{\text{AllI}} & = \frac{1}{2} \int d^2 r \left[ \Gamma_s S^2(r) S^2(r) + \Gamma_c m(r) m(r) \right], \\
  \delta \sigma_{xx}^{\text{AllII}} & = \frac{1}{2} \int d^2 r \Gamma_c m(r) m(r).
\end{align*} \]

III. Hartree-Fock spin conductivity

Perturbative in interactions \( \sim O(\Gamma_{ss}) \)
\[ \delta \sigma_{xx}^{\text{H}} = 0 \text{ in every disorder realization!} \]

IV. Large winding number expansion

WZNW Finkel’tein nonlinear sigma model

\[ \begin{array}{c|c|c}
  \text{Class} & \text{Target manifold} G & \text{WZNW level } K \\
  \hline
  \text{CI} & \text{Sp}(2nN) & |ν|/2 \\
  \text{AllI} & \text{U}(nN) & |ν| \\
  \text{AllII} & \text{O}(nN) & |ν| \\
\end{array} \]

\( S[\hat{Q}] = S_0[\hat{Q}] + S_1[\hat{Q}] + \Gamma_k[\hat{Q}], \quad \hat{Q} \in G \)

\( S_0[\hat{Q}] \) represents the standard dynamical sigma model on the target manifold \( G \):
\[ S_0[\hat{Q}] = \frac{1}{2\pi} \int \text{Tr} \left[ \nabla \hat{Q}(r) \cdot \nabla \hat{Q}(r) \right] - \hbar \int \text{Tr} \left[ \frac{1}{2} \Omega(r) \nabla^2 \hat{Q}(r) - \nabla \hat{Q}(r) \nabla \hat{Q}(r) \right]. \]

\( S_1[\hat{Q}] \) encodes interactions \( \Gamma_{ss} \Gamma_{ss}[\hat{Q}] \) is the WZNW action at level \( K \).

One-loop RG equations for \( \lambda \) and \( \lambda_A \) \( \sim O(1/K) \):
\[ (\gamma_{ss} = 4\Gamma_{ss}/\pi h) \]
\[ \begin{array}{c|c|c|c}
  \text{Class} & \text{RG equation} & \text{At}\text{\textsc{}Altshuler-Aronov corrections} \\
  \hline
  \text{CI} & \lambda_A/\lambda = \lambda^2 \left[ 1 - (K\lambda)^2 \right] & \lambda_A = O(1/K) \\
  \text{AllI} & \lambda_A/\lambda = \lambda^2 \left[ 1 - (K\lambda)^2 \right] & \lambda_A = O(1/K) \\
  \text{AllII} & \lambda_A/\lambda = -\lambda^2 \left[ 1 - (K\lambda)^2 \right] & \lambda_A = O(1/K) \\
\end{array} \]

V. Conclusion

Spin and thermal conductances of TSC surfaces are unmodified by interactions in two limits—Universal surface transport coefficients (analogous to 2D QHE).

A simple picture: Oscillations in color and spin are forbidden by \( T \) symmetry.