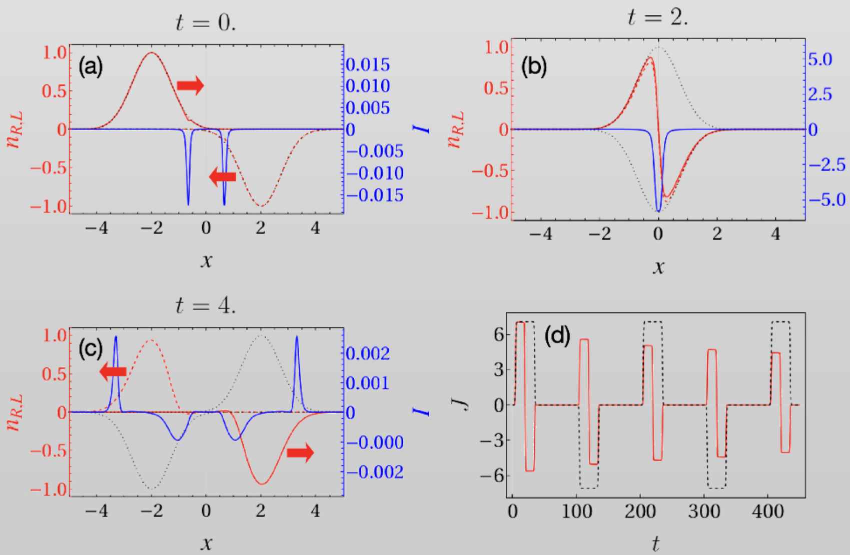
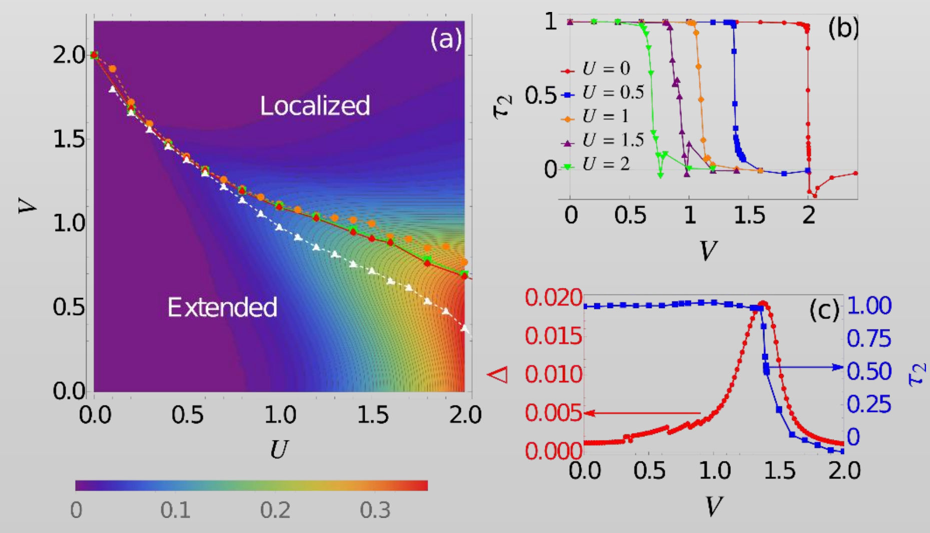
Shock and bounce edge-state dynamics, and fractal-mediated superconductivity

Matthew S. Foster
Rice University

June 12, 2023





First, an advertisement:

- In a strongly interacting fermion-boson soup*, hydrodynamic collective modes for conserved quantities (charge, spin, heat) are classical or quantum?

* Marginal Fermi liquid with Planckian dissipation and ordinary (non-SYK) impurity scattering

RICE CMTC MIT NSF THE Welch FOUNDATION

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PHYSICAL REVIEW B **106**, 155108 (2022)

Quantum interference of hydrodynamic modes in a dirty marginal Fermi liquid

Tsz Chun Wu,[1] Yunxiang Liao,[2,3] and Matthew S. Foster[1,4]

Enhancement of Superconductivity in a Dirty Marginal Fermi Liquid

Tsz Chun Wu,[1] Patrick A. Lee,[2] and Matthew S. Foster[1,3]

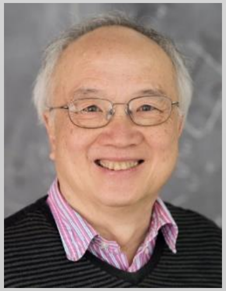
arXiv:2305.13357v1 [cond-mat.supr-con] 22 May 2023

Tsz Chun Wu

Rice University, Trexquant

Yunxiang Liao

JQI/CMTC U. Maryland, KTH



Patrick Lee

MIT

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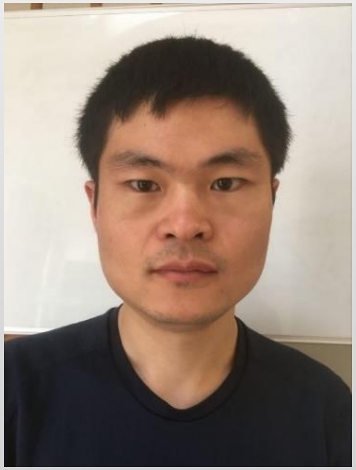

Xinghai Zhang
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1. Shock + Bounce PRL 127, 026801 (2021)
2. Fractal SC PRB 106, L180503 (2022)
3. Magnetism at the onset of a spectrum-wide quantum-critical transition (in preparation)

Dissipation in "protected" quantum systems

Nonequilibrium quantum dynamics (quench, Floquet, etc.):

- Often interested in "protected" quantum systems
  1. Integrable or effectively integrable (MBL?)
  2. Low-dimensional (kinematically constrained)
  3. Topological

- Prethermalization plateau (collisionless steady-state)

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  3. Topological
- Prethermalization plateau (collisionless steady-state)
- Thermalization in a not-quite ideal quantum system
  1. Frequently mediated by "irrelevant" operators
  2. Hard to deal with by otherwise powerful (e.g. field theory) methods
- Alternative: Semiclassical (?) hydrodynamics

Light-pulse induced dynamics in a helical edge loop

- **1D helical (spin-momentum locked) edge liquid**

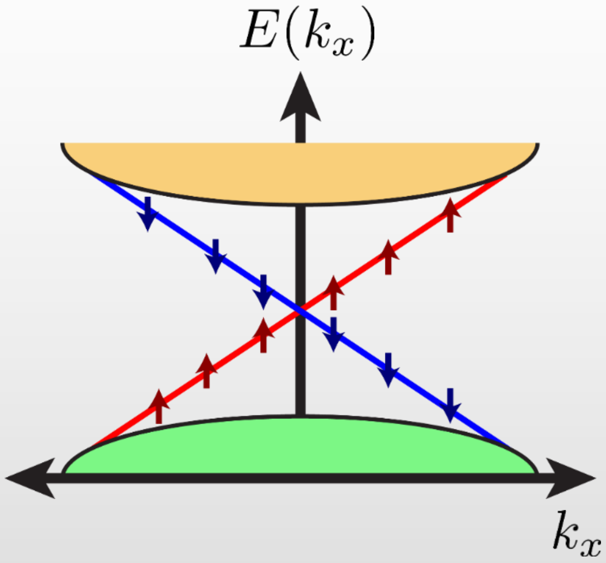
  1. **Edge states of a 2D Z_2 topological insulator**
  2. **Synthetic realizations, e.g.**

  – ***N*-layer graphene**

  Abanin, Lee, Levitov 2006
  Young, Jarillo-Herrero *et al.* 2014
  Che, Lau, Murthy, Fertig *et al.* 2020

  – **Edge liquids of higher-order TIs**

  Schindler, Yazdani, Bernevig, Neupert *et al.* 2018
  Shumiya, Balicas, Zhang, Yao, Hasan *et al.* 2022



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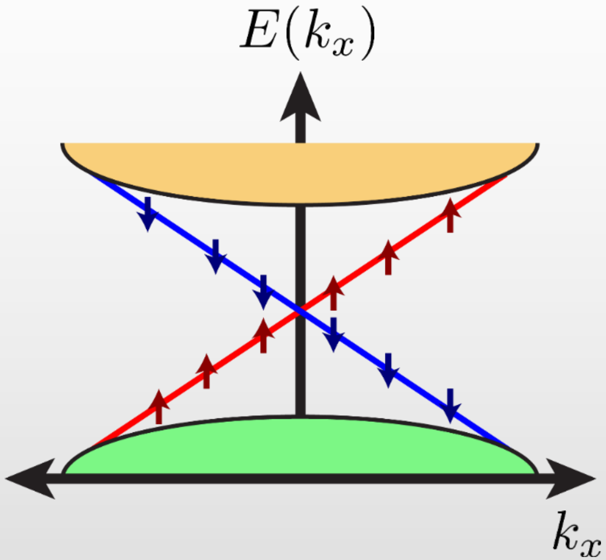
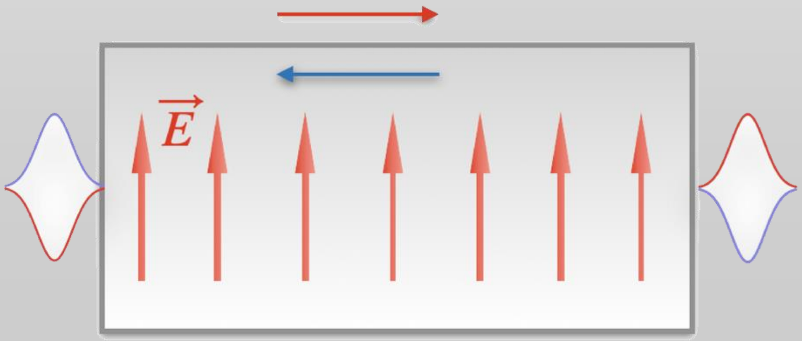
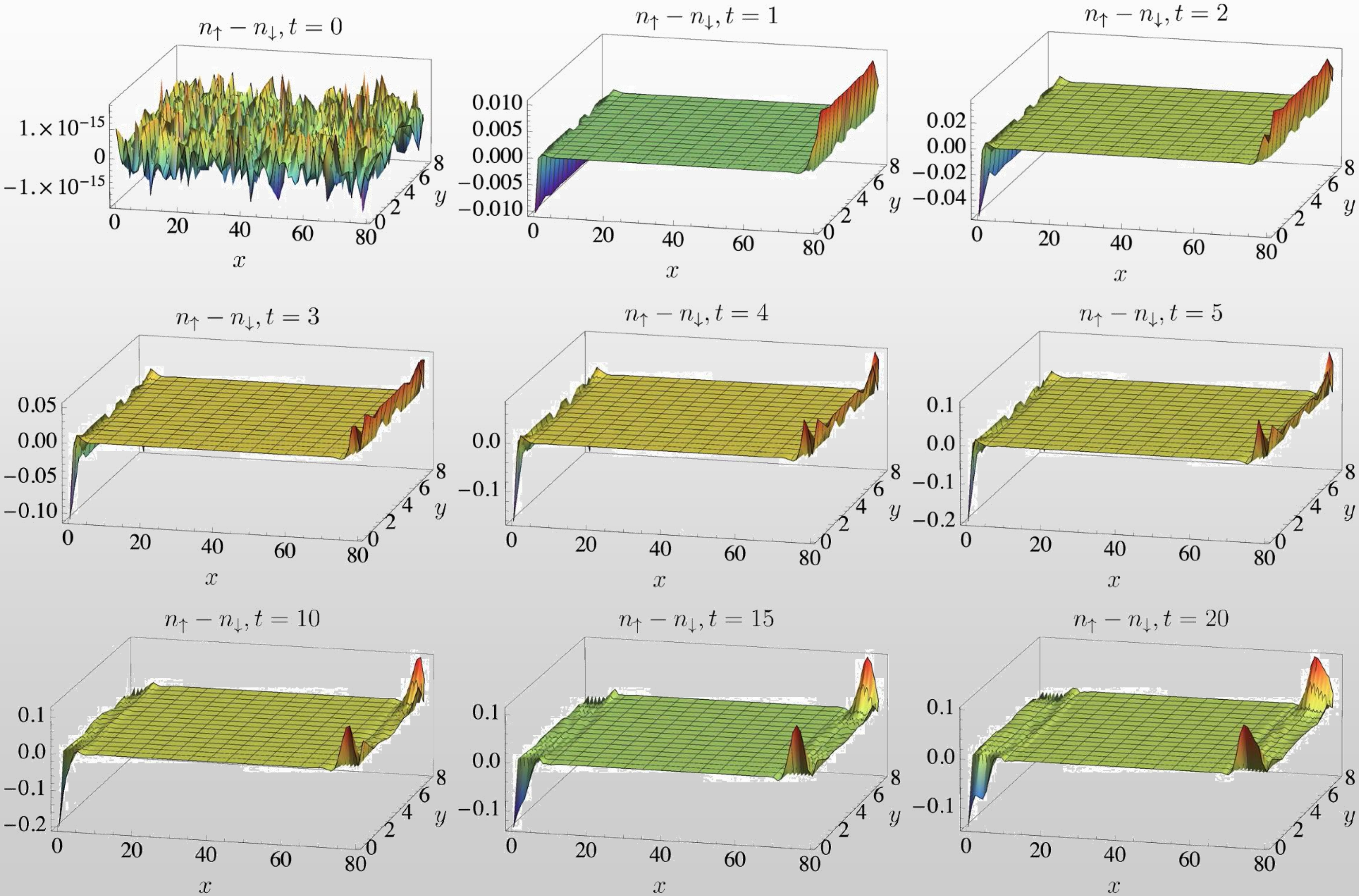


- **Setup: Light-induced (axial anomaly) circulating spin, charge packets**



Spin and charge packets (ideal edges)

- **Electric pulse-induced “quench” (non-interacting Kane-Mele model, open BC)**



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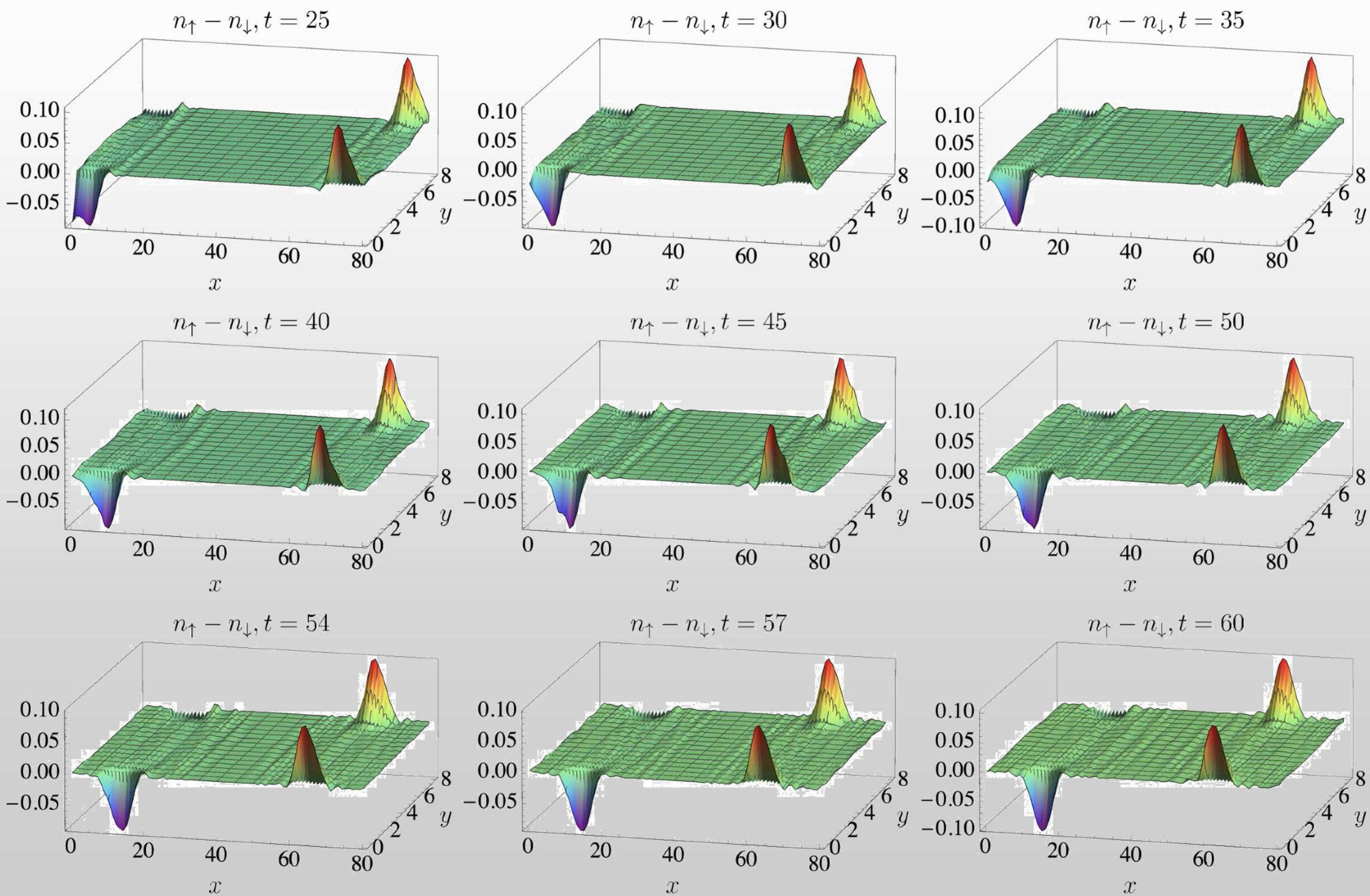

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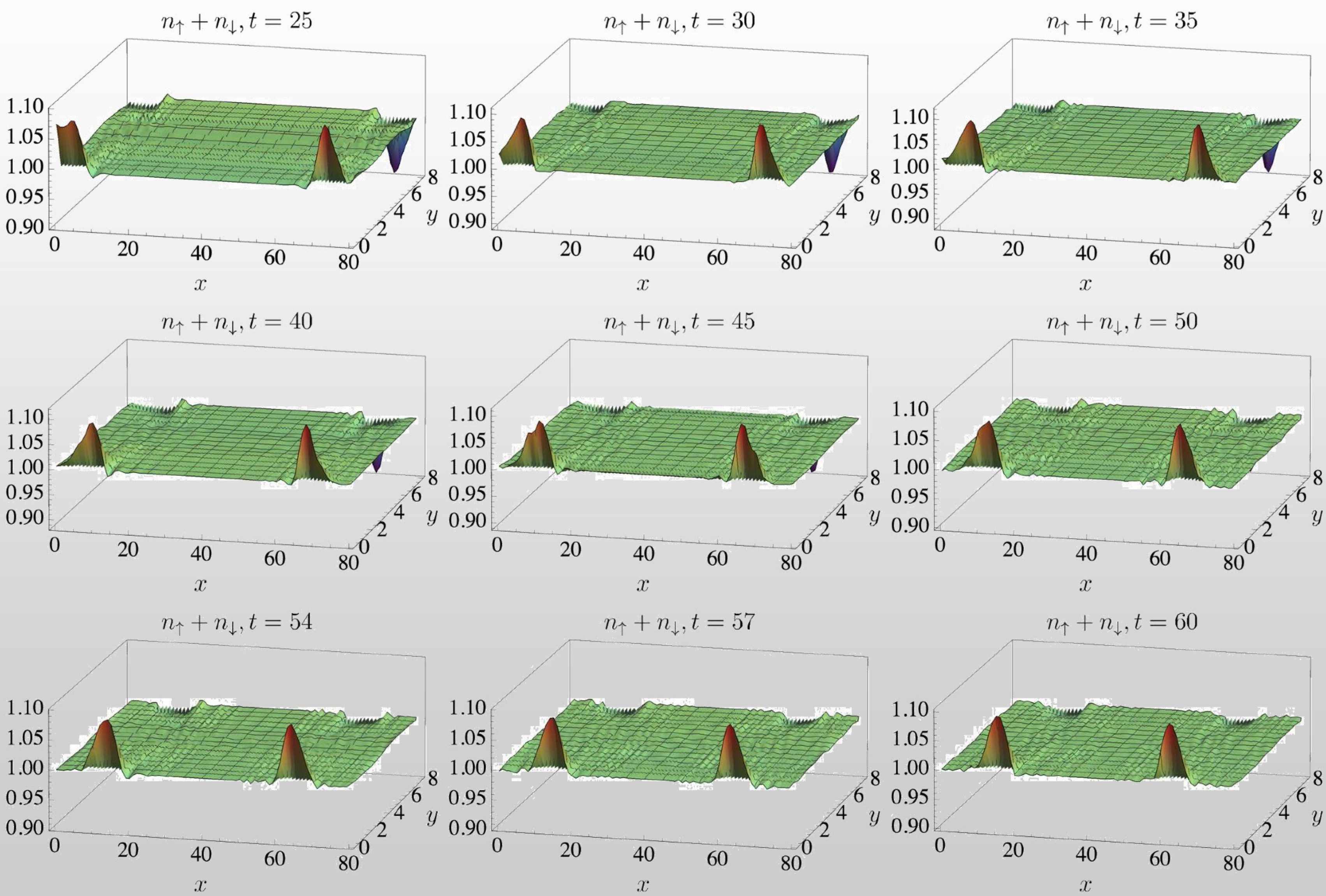
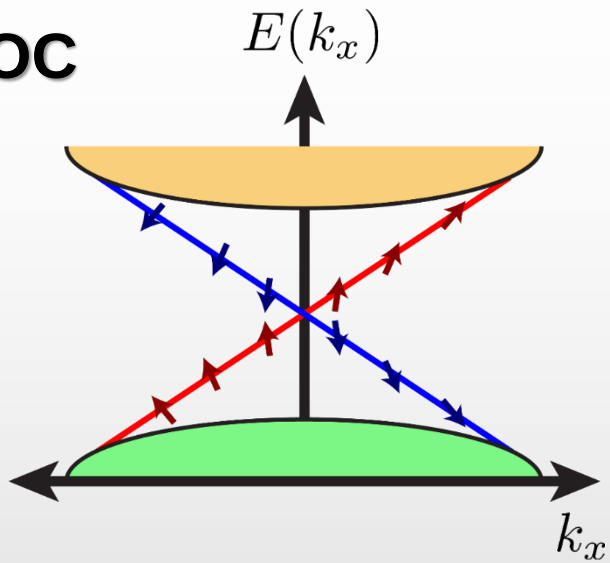
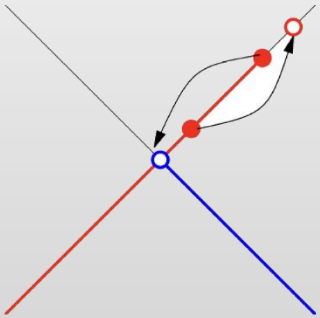


Light-pulse induced dynamics in a helical edge loop

- **1D helical edge liquid with Rashba SOC**
  - **Slow rotation of spin texture with k_x**
- **Rashba SOC + Screened Coulomb:**
  - **Irrelevant "1-particle Umklapp" (1PU) scattering processes**





Lezmy, Oreg, Berkooz 2012
Kainaris, Gornyi, Carr, Mirlin 2014
Chou, Levchenko, Foster 2015

Right + Right ⇨ Right (sum) + Left (zero) momentum

$$H_{1\mathrm{PU}} = W \int dx \left[R^\dagger \, R \, L^\dagger(-i\partial_x)R + L^\dagger \, L \, R^\dagger(-i\partial_x)L + \mathrm{H.c.}\right]$$

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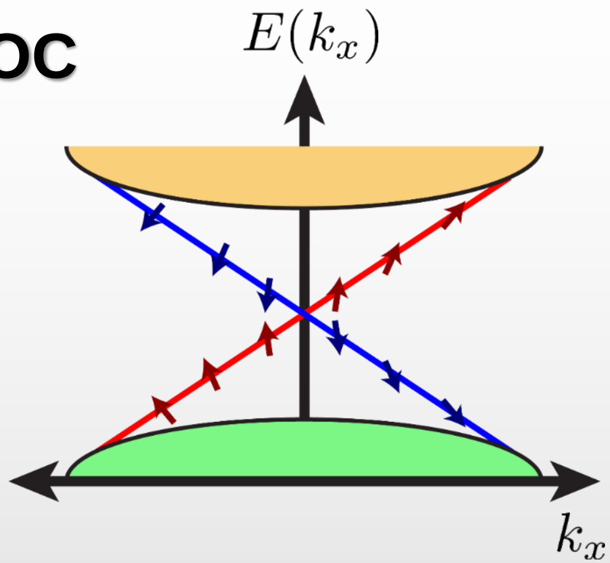

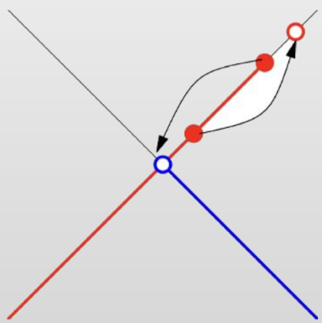

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- **Luttinger liquid interactions?**
  - **Bosonize, refermionize to remove them**
  - **Price: 1PU interaction becomes nonlocal due to "strings"**

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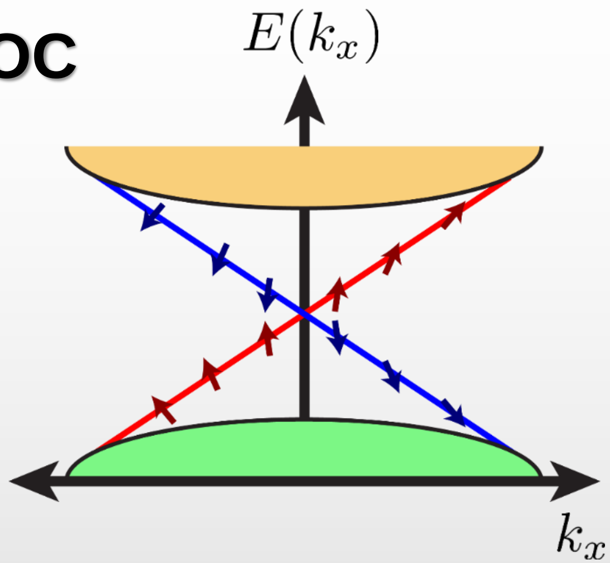

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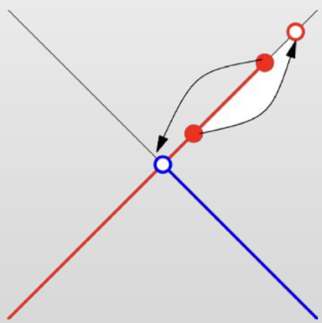

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- **Effect on circulating charge packets?**

Chiral hydrodynamics

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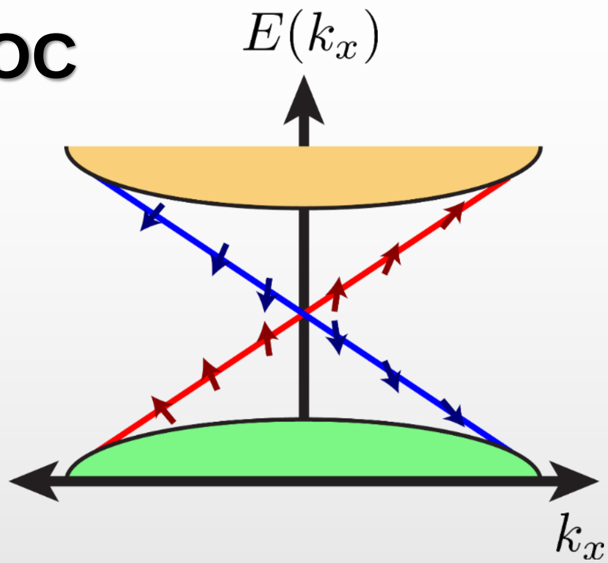

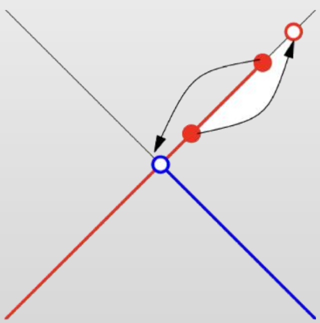

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Particles: $$\partial_t n_{R,L} \pm v_F \partial_x n_{R,L} = \pm \frac{eE}{h} \pm I$$

Momenta: $$\partial_t P_{R,L} \pm v_F \partial_x P_{R,L} = eE\, n_{R,L} + F_{\text{in}}^{R,L}$$

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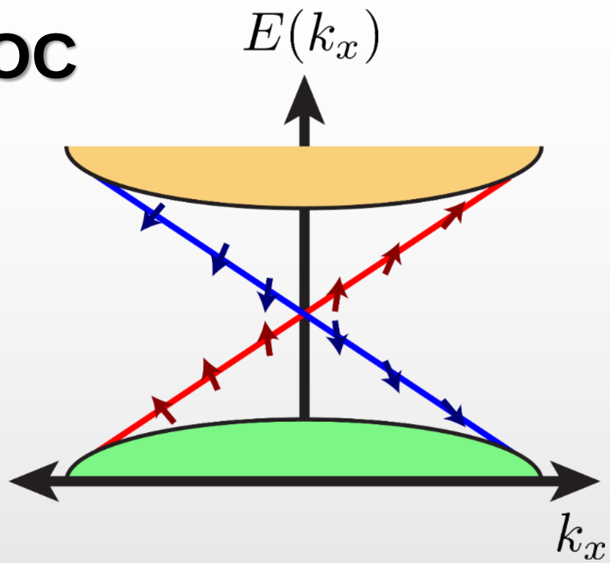

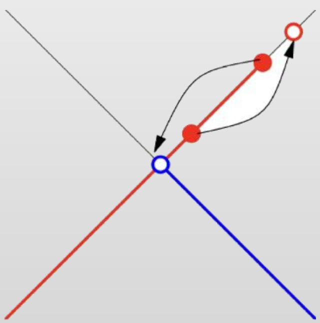

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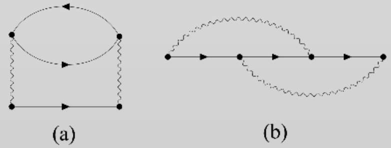
<u>After external field is turned off, no disorder:</u>

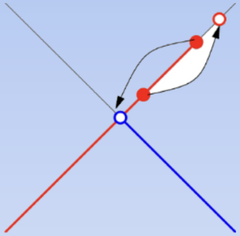
Particles: $$\partial_t n_{R,L} \pm v_F \partial_x n_{R,L} = \pm I$$



Momenta: $$\partial_t P_{R,L} \pm v_F \partial_x P_{R,L} = 0$$

Right, left momenta separately conserved (as in SG model)

Shock dynamics in a helical edge loop



Right + Right ⟹ Right (sum) + Left (zero) momentum

After external field is turned off, no disorder:

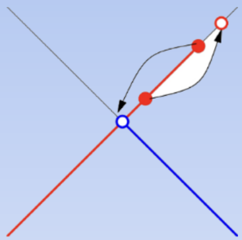
Particles: $$2\,\partial_{\pm}\,n_{R,L} = \pm I(n_R, n_L, T_R, T_L)$$

Momenta: $$2\,\partial_{\pm}\,P_{R,L} = 0$$

Lightcone coordinates
$x_{\pm} = x \pm t, \quad v_F \equiv 1$

Local initial right-mover excess: $$n_R^{(0)} = n_0\,\exp\left(-x^2/\xi^2\right)$$

Shock dynamics in a helical edge loop



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Local right-mover (left-mover) temperature heated (cooled) by imbalance relaxation *I*

$$T_R(t,x) = \sqrt{T_0^2 + 12\left\{\left[n_R^{(0)}(x_-)\right]^2 - \left[n_R(t,x)\right]^2\right\}}$$

$$T_L(t,x) = \sqrt{T_0^2 - 12\left[n_L(t,x)\right]^2}$$

Shock dynamics in a helical edge loop

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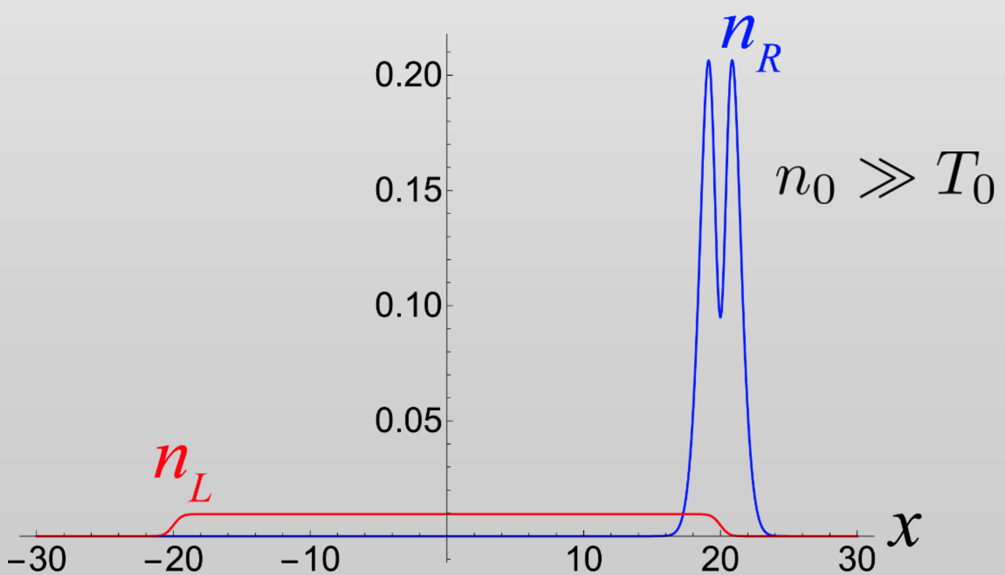
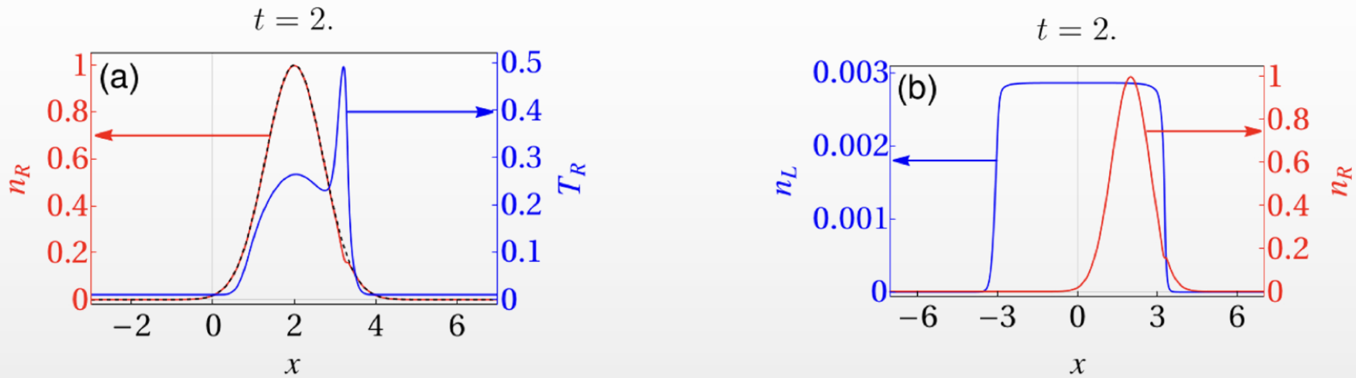
Short-time perturbation theory:



Shock dynamics in a helical edge loop

Shock formation: **Imbalance self-focuses at leading right-edge**



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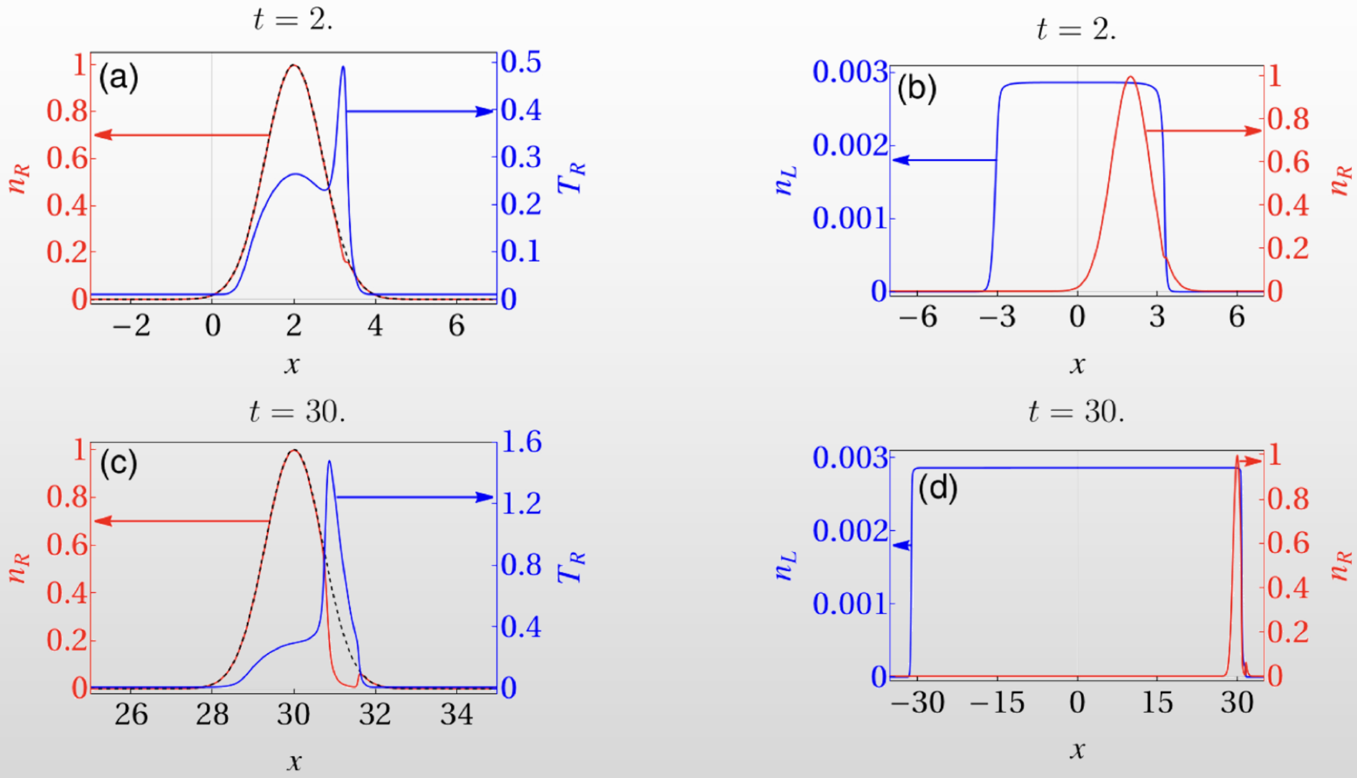

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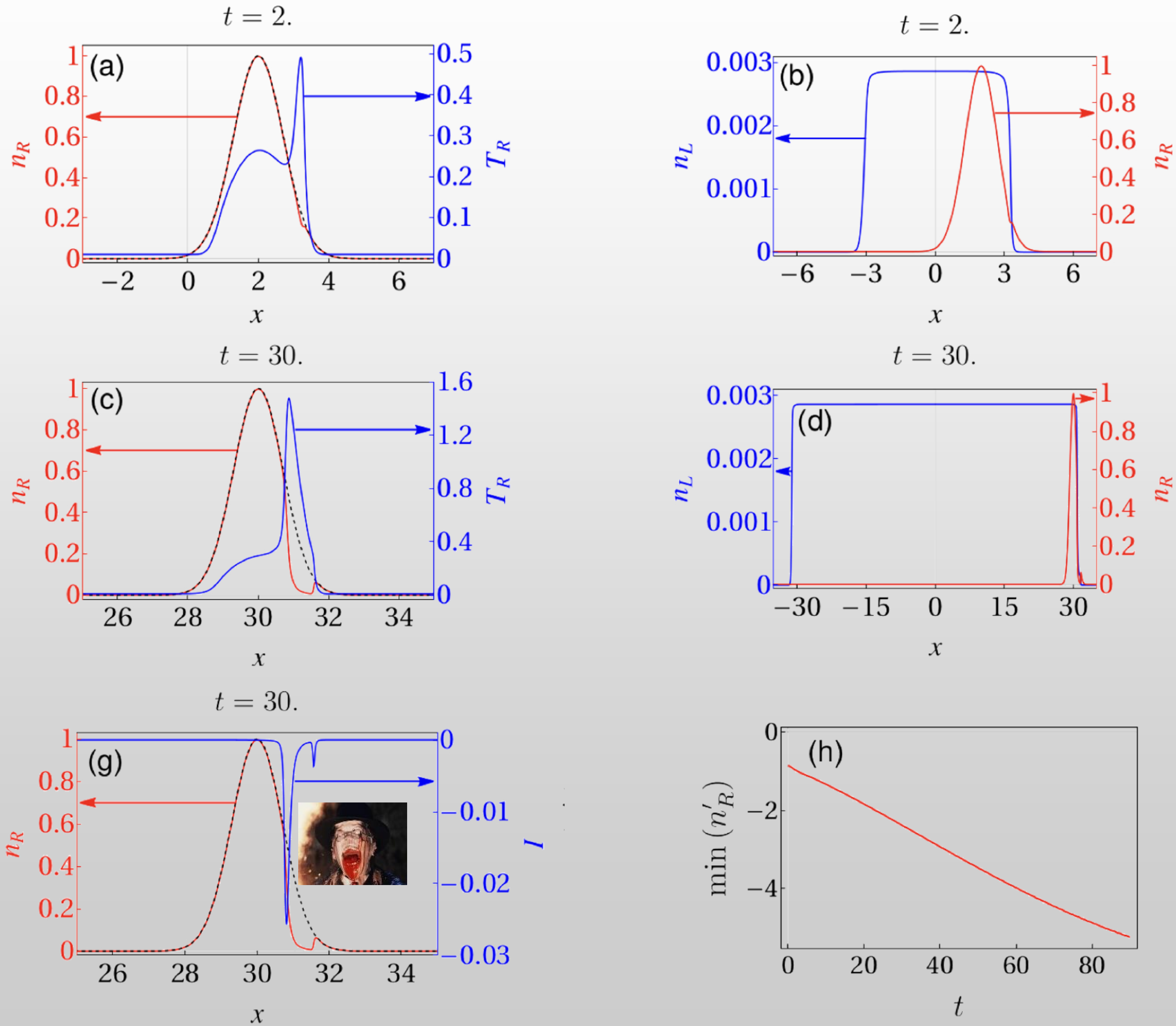
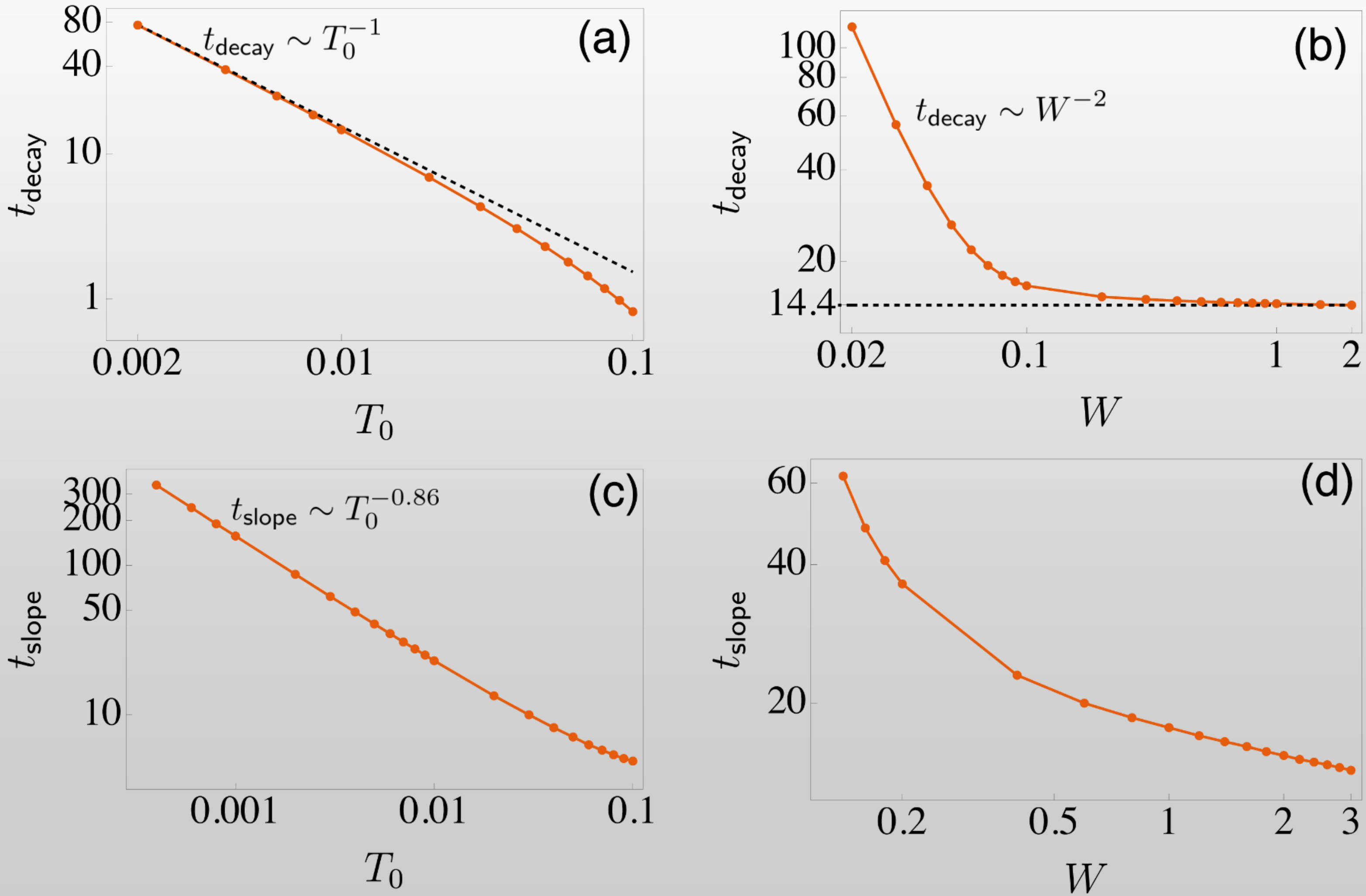


Shock dynamics in a helical edge loop

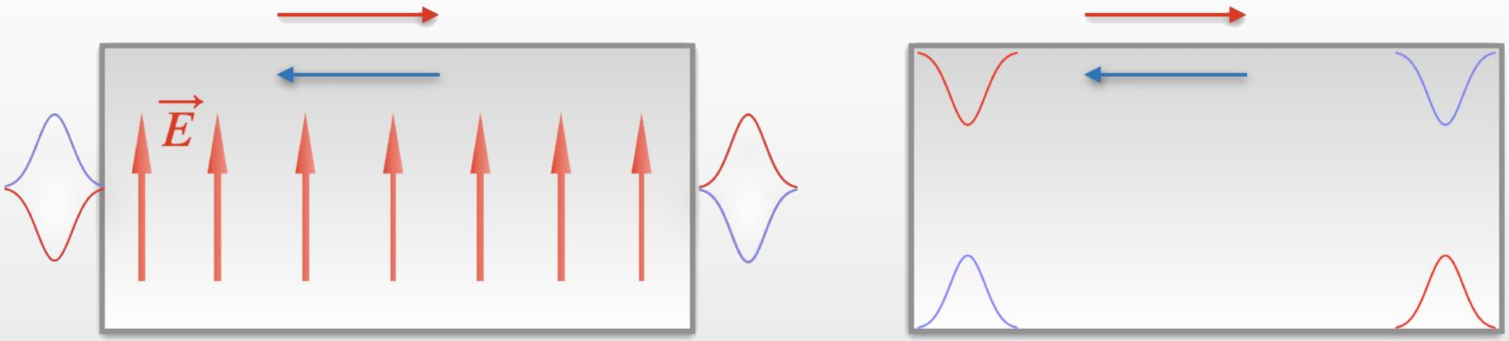
$$\Delta N_R(t)/N \propto t/t_{\text{decay}}$$

$$\min n'_R(t) \simeq -(n_0/\xi)(1 + t/t_{\text{slope}})$$

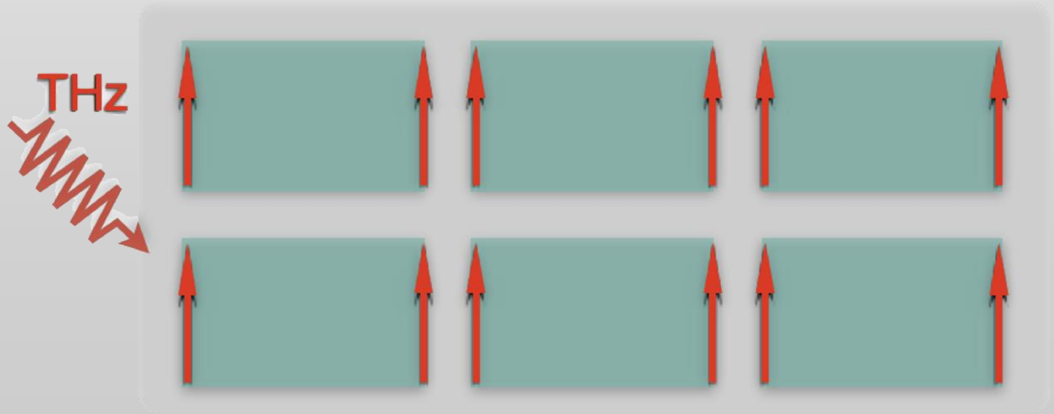


Bounce dynamics between opposite charge packets

- 4 light-induced circulating spin, charge packets

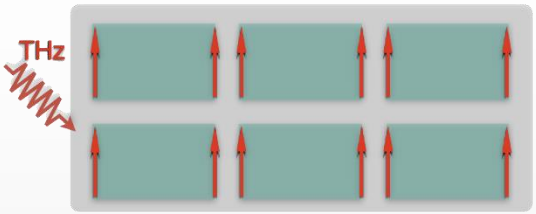


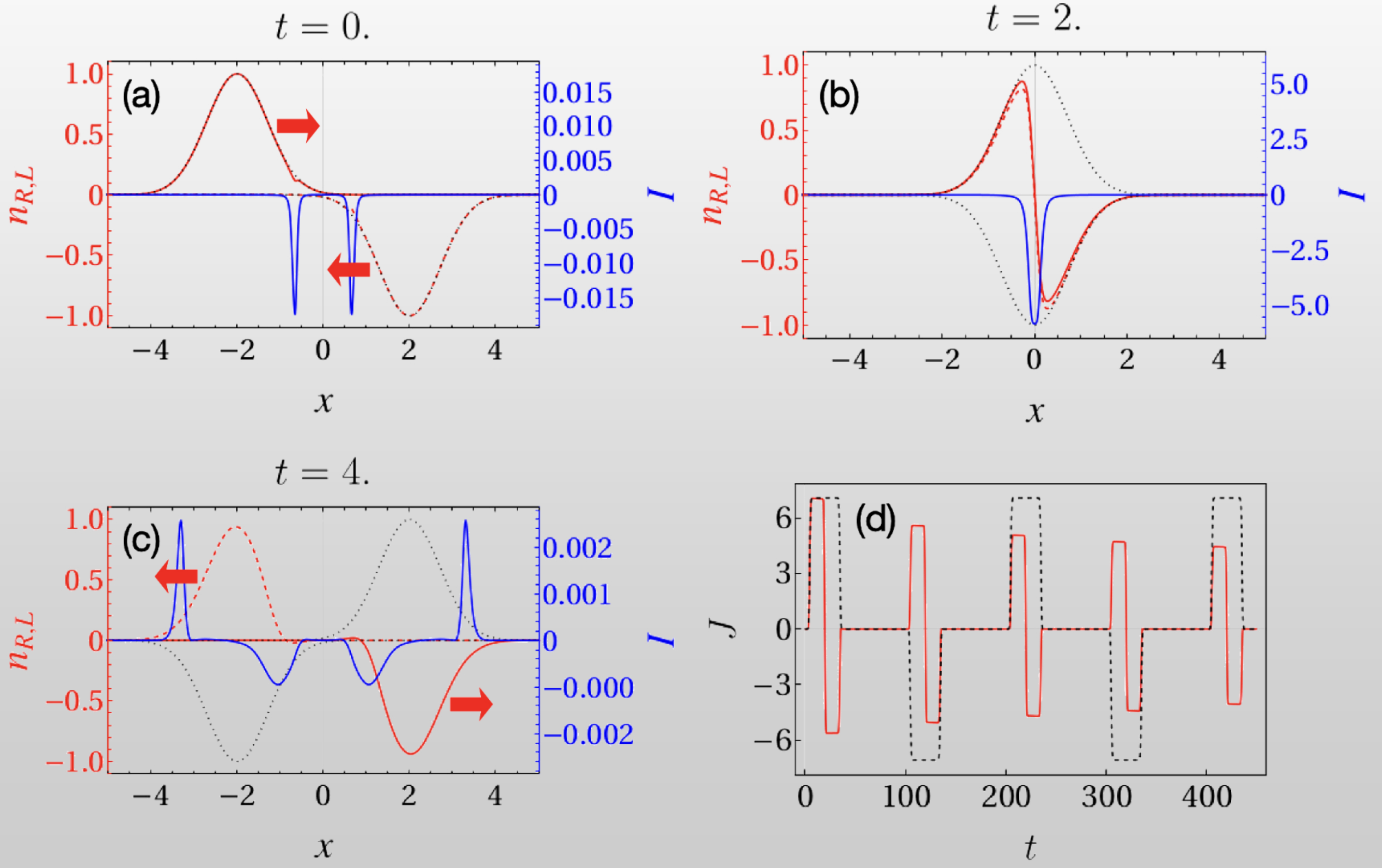
- Current oscillations along field: “THz antenna array”



Bounce dynamics between opposite charge packets

- **Imbalance collision-dominated regime: frequency doubling (shock and bounce)**





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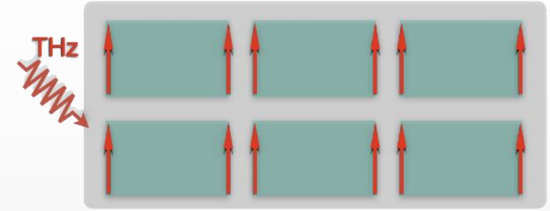

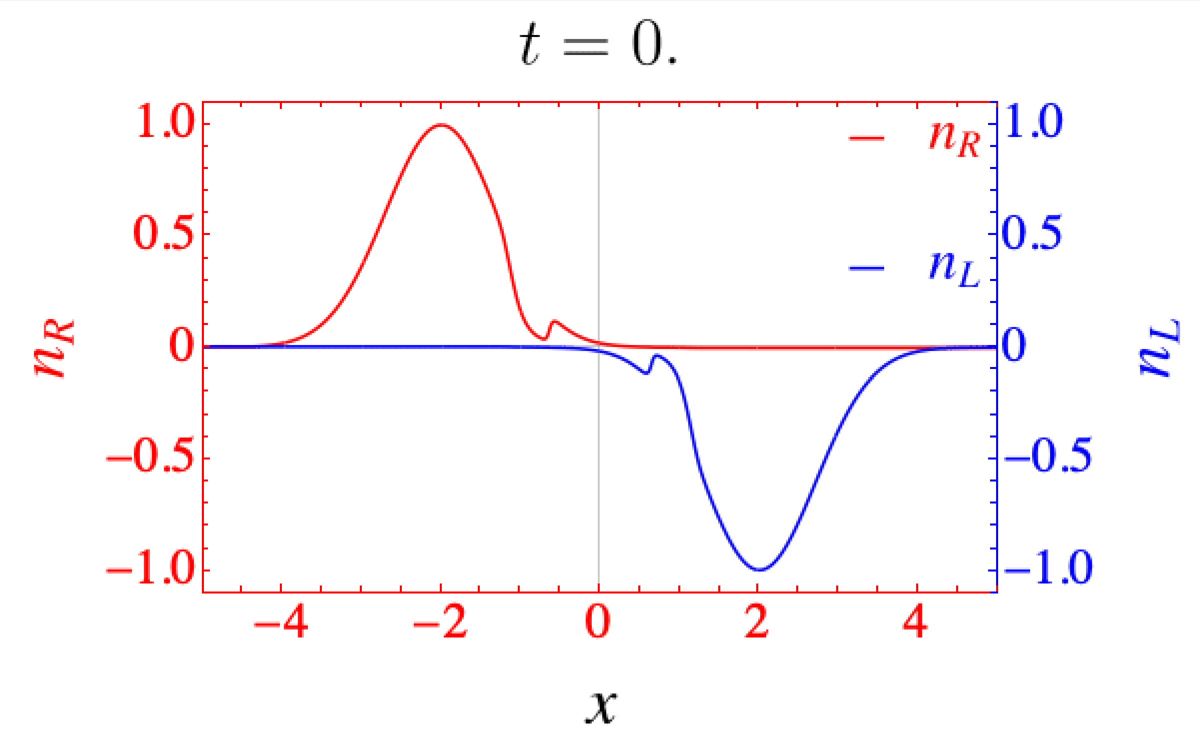

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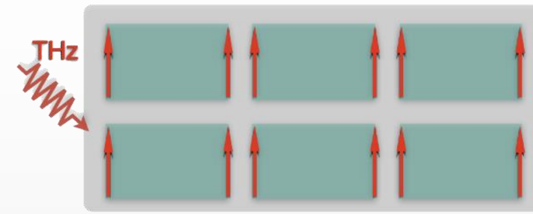

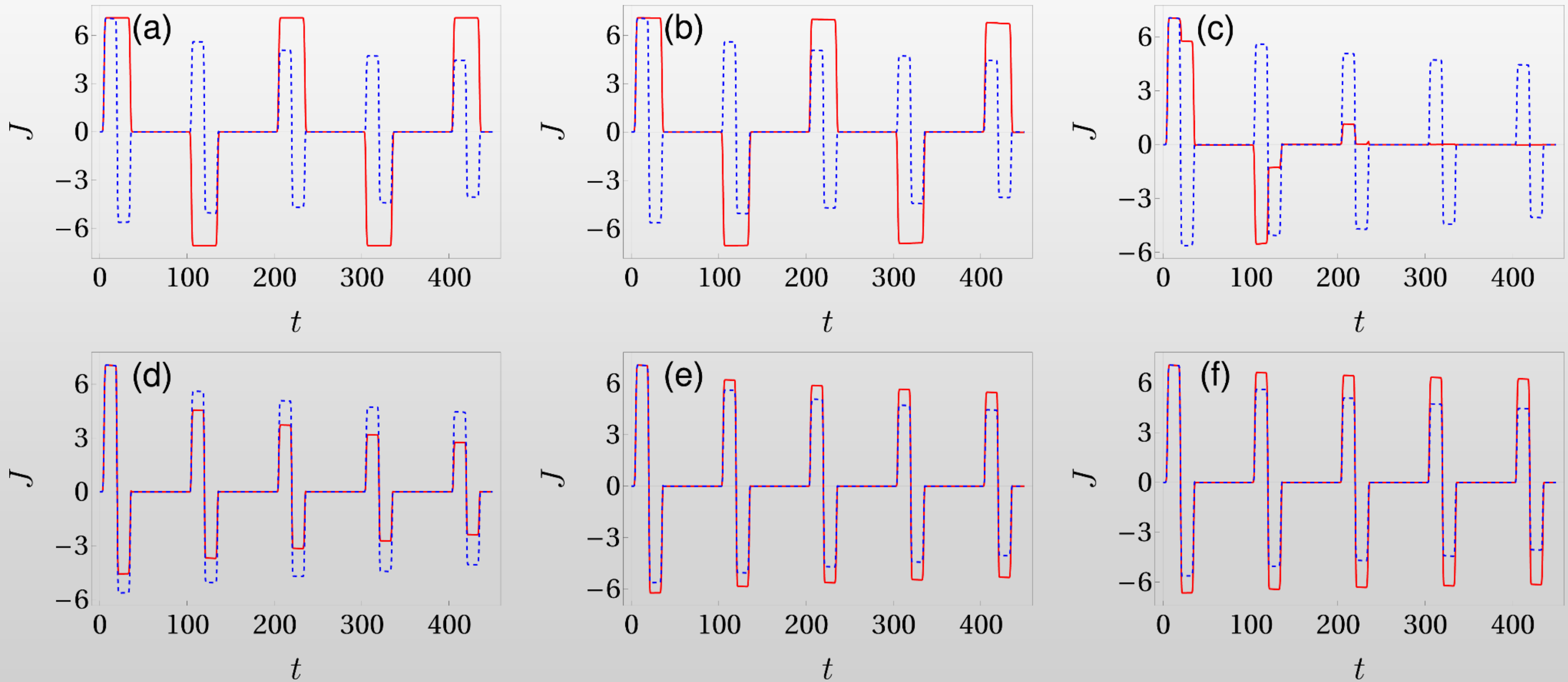

Dimensionless interaction strengths (blue dashed is W = 1):

(a) W = 0 (b) W = 0.01 (c) W = 0.1 (d) W = 0.5 (e) W = 2 (f) W = 5

fractal-mediated superconductivity

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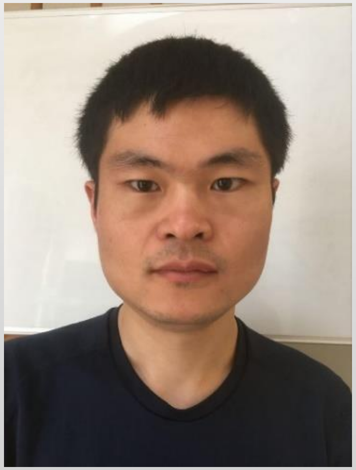

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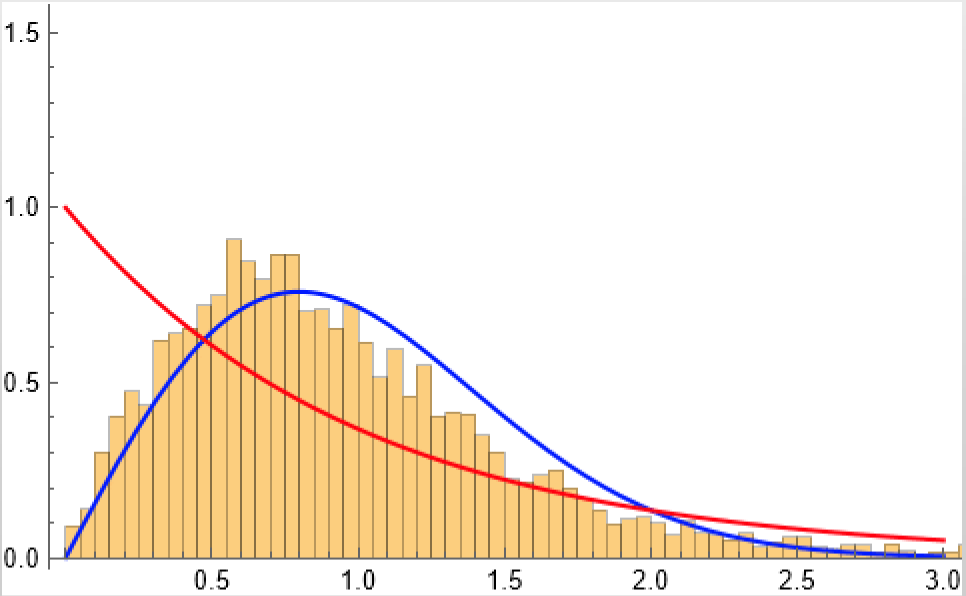
Warmup: Diagnostics for Anderson localization

Noninteracting particles with a random Hamiltonian*

$$H = \sum_{i,j} h_{ij}\, c_i^\dagger\, c_j$$

1. **Level spacing statistics** $s_i = \dfrac{\varepsilon_{i+1} - \varepsilon_i}{\Delta\varepsilon}$

$P(s)$



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* Non-interacting PRBM model, $1/2 < \alpha < 3/2$, $N = 4000$

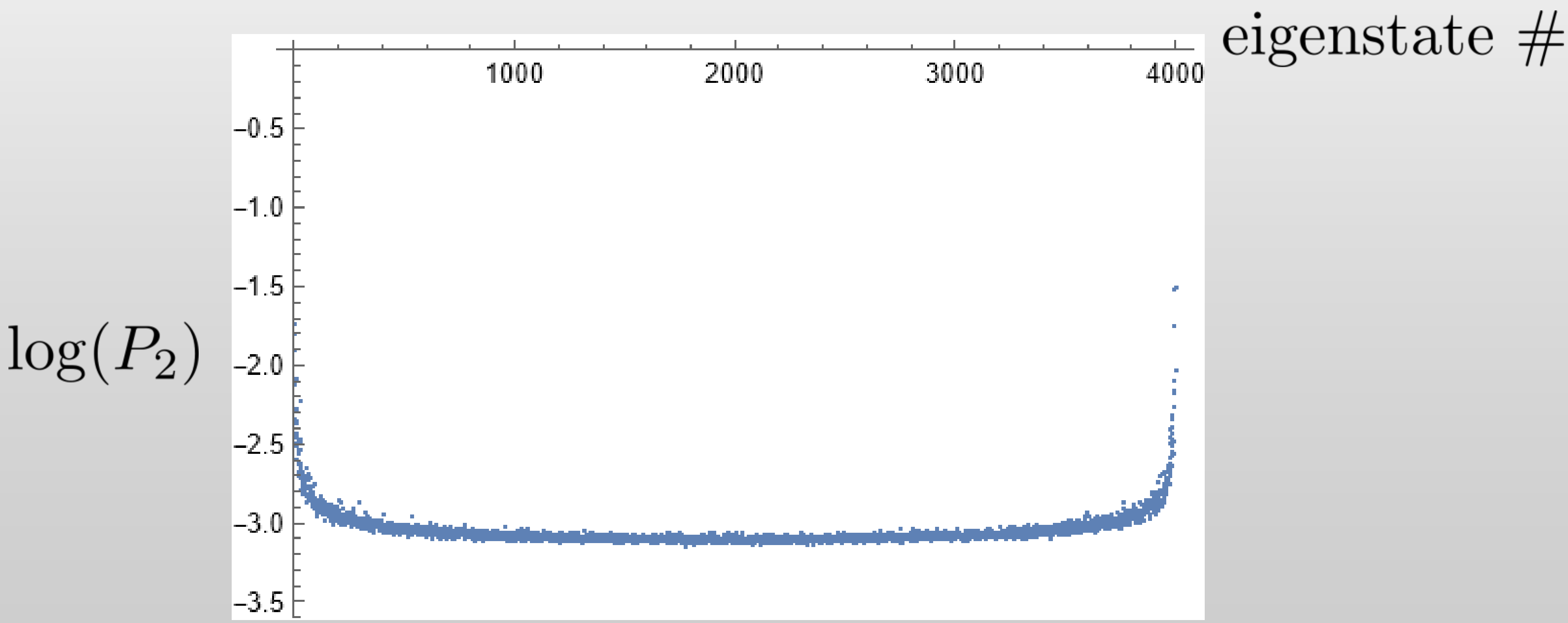
Review:
Evers and Mirlin RMP 2008

Warmup: Diagnostics for Anderson localization

Noninteracting particles with a random Hamiltonian*

$$H = \sum_{i,j} h_{ij}\, c_i^\dagger\, c_j$$

2. Inverse participation ratio $P_2(\varepsilon) = \sum_i |\psi_\varepsilon(x_i)|^4$



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Review:
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Warmup: Diagnostics for Anderson localization

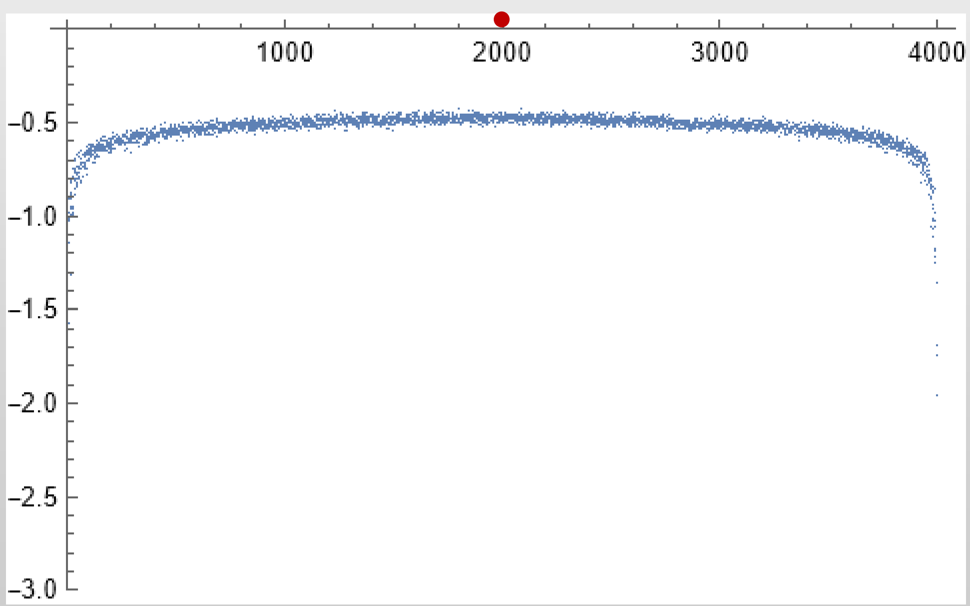
3. Chalker-scaling correlator (energy-split IPR)

Chalker and Daniell 1988
Chalker 1990

$$C_{\varepsilon_1,\varepsilon_2} \equiv \frac{\sum_i |\psi_{\varepsilon_1}(x_i)|^2 |\psi_{\varepsilon_2}(x_i)|^2}{\frac{1}{2}\left(\sum_i |\psi_{\varepsilon_1}(x_i)|^4 + \sum_i |\psi_{\varepsilon_2}(x_j)|^4\right)}$$

eigenstate ε_2 #

$\log(C_{\varepsilon_1=0,\varepsilon_2})$



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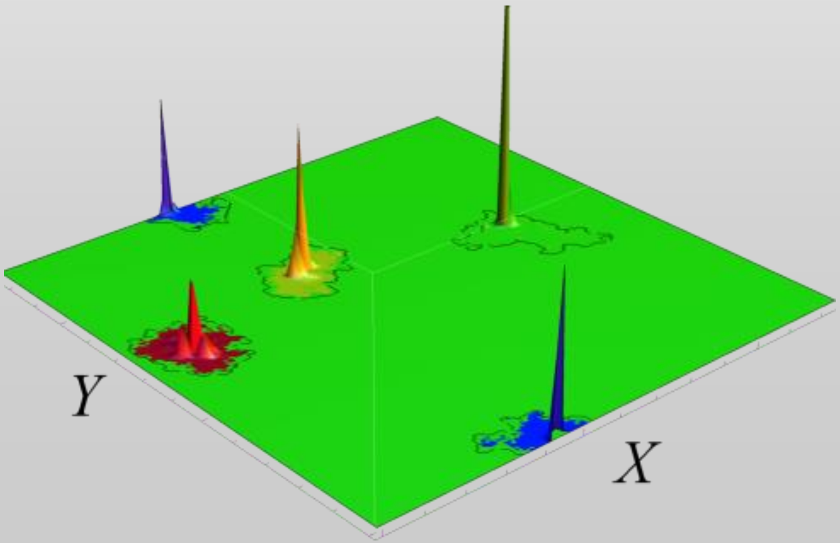
Nearby states in energy:

- **Vanishing overlap in Anderson insulator** $C_{\varepsilon_1,\varepsilon_2} \sim \delta_{\varepsilon_1,\varepsilon_2}$



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Chalker 1990

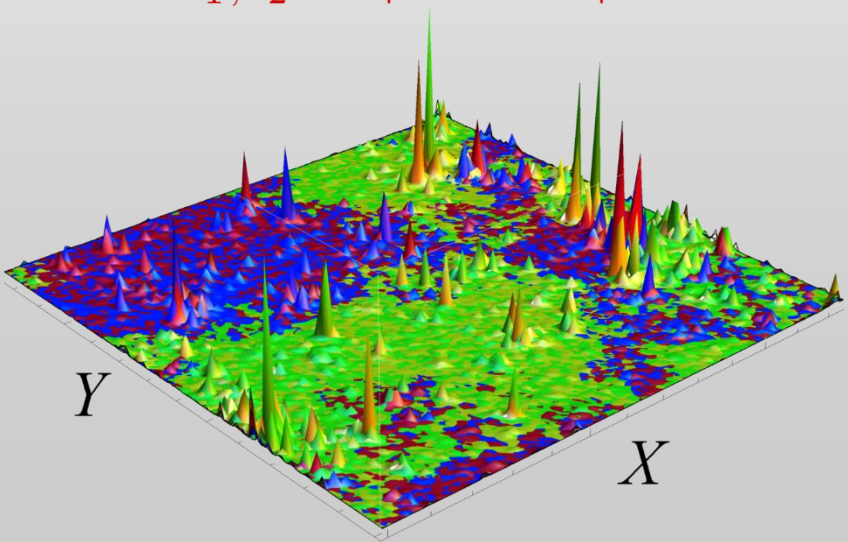
$$C_{\varepsilon_1,\varepsilon_2} \equiv \frac{\sum_i |\psi_{\varepsilon_1}(x_i)|^2 |\psi_{\varepsilon_2}(x_i)|^2}{\frac{1}{2}\left(\sum_i |\psi_{\varepsilon_1}(x_i)|^4 + \sum_i |\psi_{\varepsilon_2}(x_j)|^4\right)}$$

Nearby states in energy:

- **Vanishing overlap in Anderson insulator** $C_{\varepsilon_1,\varepsilon_2} \sim \delta_{\varepsilon_1,\varepsilon_2}$
- **Uniform overlap in diffusive metal**
- **Fractal overlap at the localization transition** $C_{\varepsilon_1,\varepsilon_2} \sim |\varepsilon_1 - \varepsilon_2|^{-\frac{d-d_2}{d}}$

$$P_2(\varepsilon) = \sum_i |\psi_\varepsilon(x_i)|^4 \sim \left(\frac{a}{L}\right)^{d_2}$$

Fractal dimension $0 < d_2 < d$



Fractal enhancement of interactions near a MIT

Generalized Hubbard chain

$$H = \sum_{i,j,\sigma} h_{ij}\, c^{\dagger}_{i,\sigma}\, c_{j,\sigma} + U \sum_{i} c^{\dagger}_{i,\uparrow}\, c_{i,\uparrow}\, c^{\dagger}_{i,\downarrow}\, c_{i,\downarrow}$$

Interaction matrix elements in fixed realization of disorder

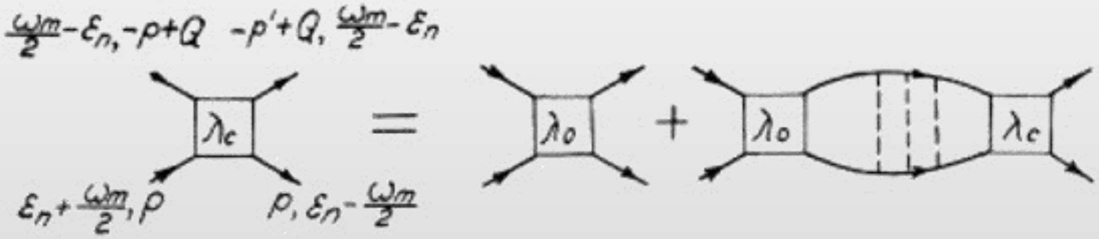
$$M_{mn} = U L^{d} \int_{x} |\psi_{m}(x)|^{2}\, |\psi_{n}(x)|^{2} \sim U \begin{cases} 1\,, & \text{metal}\,, \\ L^{d-\tau_{2}} C_{mn}\,, & \text{critical}\,, \\ (L/\xi)^{d} C_{mn} & \text{insulator} \end{cases}$$

- **Vanishing overlap in Anderson insulator** $C_{mn} \sim \delta_{m,n}$
- **Fractal overlap at the localization transition** $C_{mn} \sim |\varepsilon_{m} - \varepsilon_{n}|^{-\frac{d-d_{2}}{d}}$

∴ Interactions can be strongly enhanced near the MIT

Disorder in s-wave superconductors

- **Anderson's theorem (1960)**
  - **s-wave superconductivity is immune to non-magnetic disorder**
  - **T_c remains unchanged**



$$T_c \sim e^{-1/|U|\nu_0}$$

Review:
Altshuler and Aronov 1985

Disorder in s-wave superconductors

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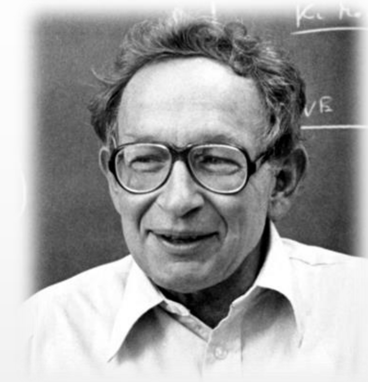
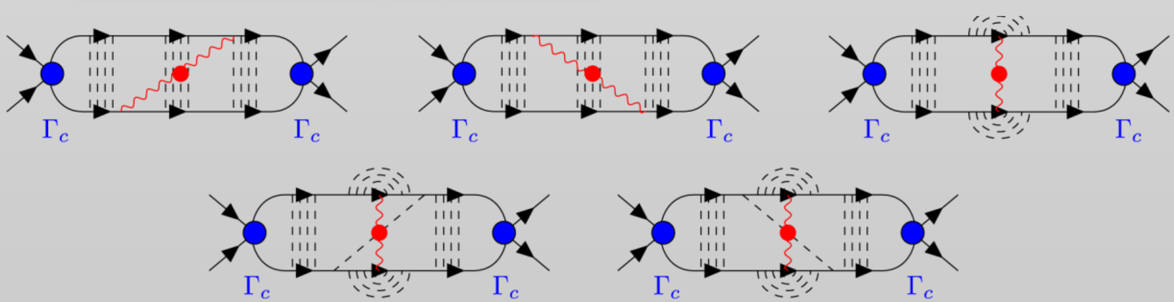


- **Maekawa & Fukuyama (1982)**

How about Anderson localization?

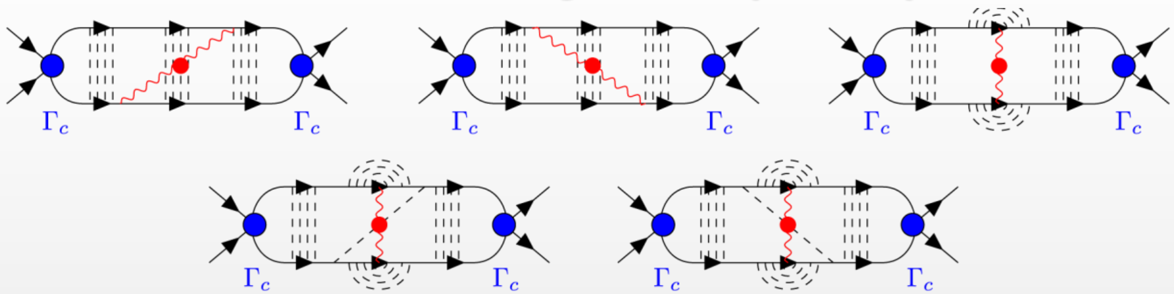


$$\frac{\delta T_c}{T_c} \sim -\ln^3\left(\frac{\Lambda}{T_c}\right)$$

Maekawa and Fukuyama 1982
Finkel'stein 1987

Multifractal enhancement in s-wave superconductors

- **Maekawa & Fukuyama (1982)**

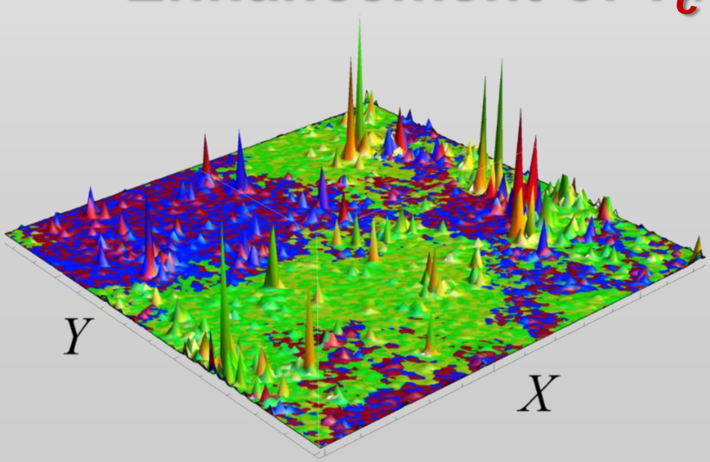


$$\frac{\delta T_c}{T_c} \sim -\ln^3\left(\frac{\Lambda}{T_c}\right)$$

Maekawa and Fukuyama 1982
Finkel'stein 1987

  - **Suppression due to quantum interference and long-ranged Coulomb interactions (SIT precursor)**

- ***Short-ranged,* other interactions + Chalker scaling:**
  Enhancement of T_c near Anderson MIT



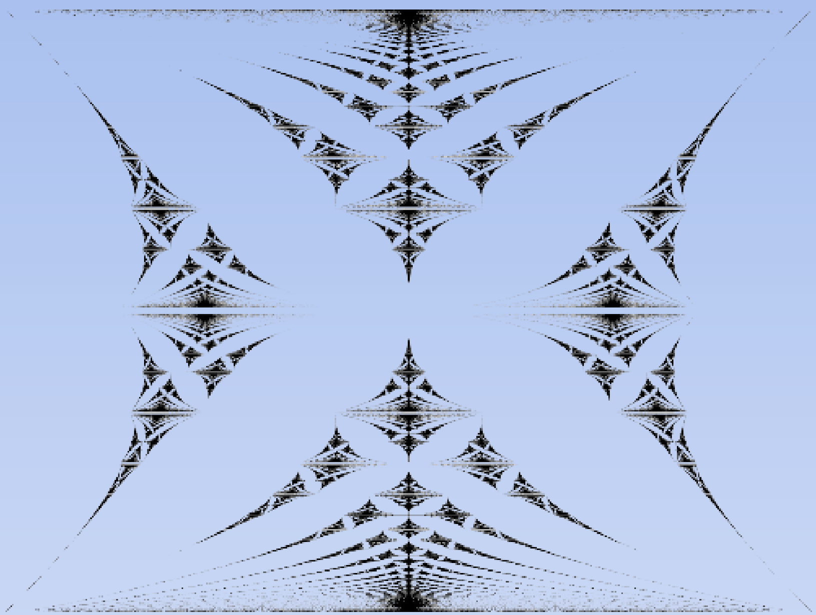
$$T_c \sim \frac{1}{\nu_0}\left(|U|\nu_0\right)^{\frac{d}{d-d_2}}$$

Feigel'man, Ioffe, Kravtsov, Yuzbashyan 2007
Feigel'man, Ioffe, Kravtsov, Cuevas 2010
Burmistrov, Gornyi, Mirlin 2012, 2015
Mayoh and Garcia-Garcia 2015
Fan and Garcia-Garcia 2020
Fan, Chern, Lin 2021
Stosiek, Evers, Burmistrov 2021

Spectrum-wide fractality and superconductivity

- **Aubry-Andre model: Uniform hopping in an incommensurate periodic potential**

$$H = -t \sum_{i\sigma} \left(c^{\dagger}_{i\sigma} c_{i+1\sigma} + c^{\dagger}_{i+1\sigma} c_{i\sigma} \right) + \sum_{i} \left[V \cos\left(2\pi \beta_p i\right) - \mu \right] n_i$$



- **Extended, localized, critical phases**

- **Critical phase (with Hofstadter butterfly spectrum) shows SWQC***

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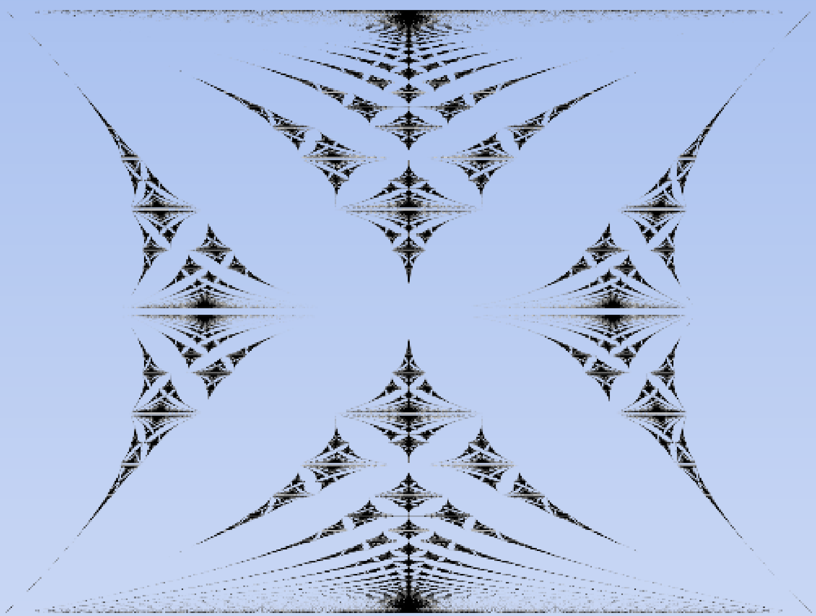

- **Extended, localized, critical phases**
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* Spectrum-wide quantum criticality

- Entire (or most) of single-particle energy spectrum consists of quantum-critical multifractal wave functions
- Occurs at fine-tuned MIT in 1D Aubry-Andre and power-law random-banded (PRBM) matrix models
- Also appears in models of surface states for bulk topological superconductors and 2D nodal (d-wave) superconductors

Ghorashi, Liao, Foster 2018
Sbierski, Karcher, Foster 2020
Ghorashi, Karcher, Davis, Foster 2020
Karcher and Foster 2021

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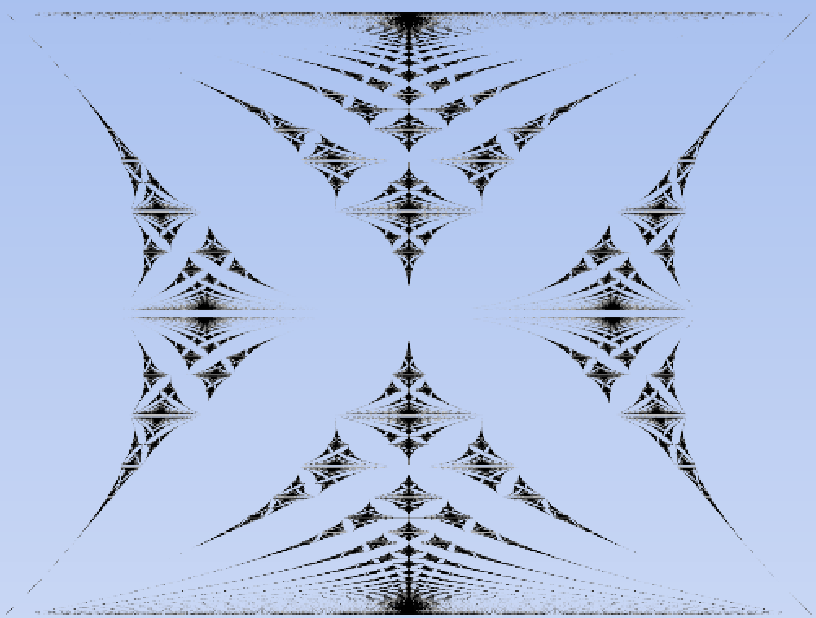

- **Extended, localized, critical phases**
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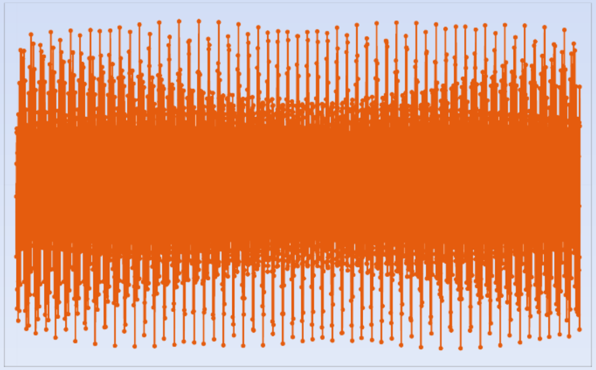
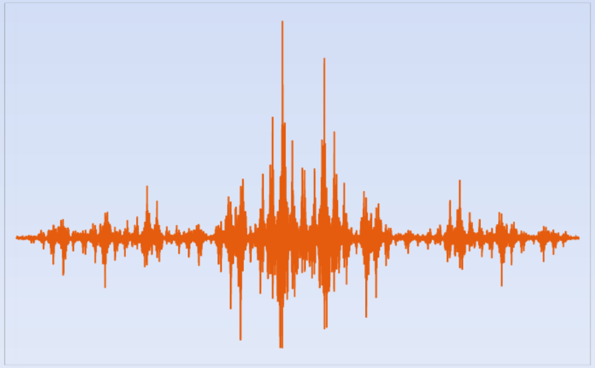
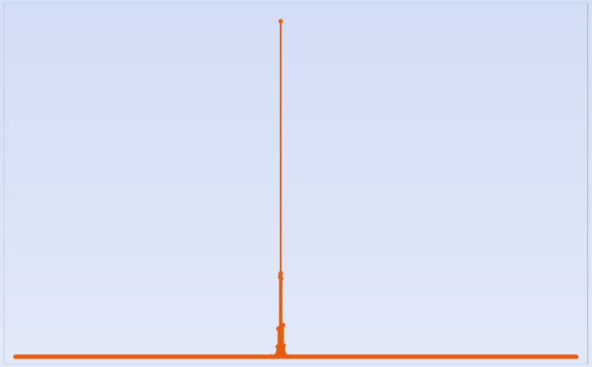
$V < 2t$



$V = 2t$



$V > 2t$



Spectrum-wide fractality and superconductivity

- **Aubry-Andre model: Uniform hopping in an incommensurate periodic potential**
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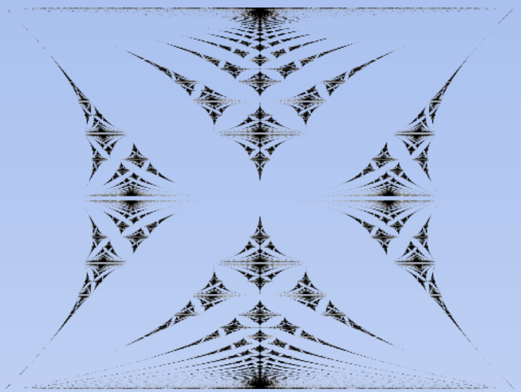


- **Self-consistent static mean-field numerics for the Aubry-Andre-Hubbard model**

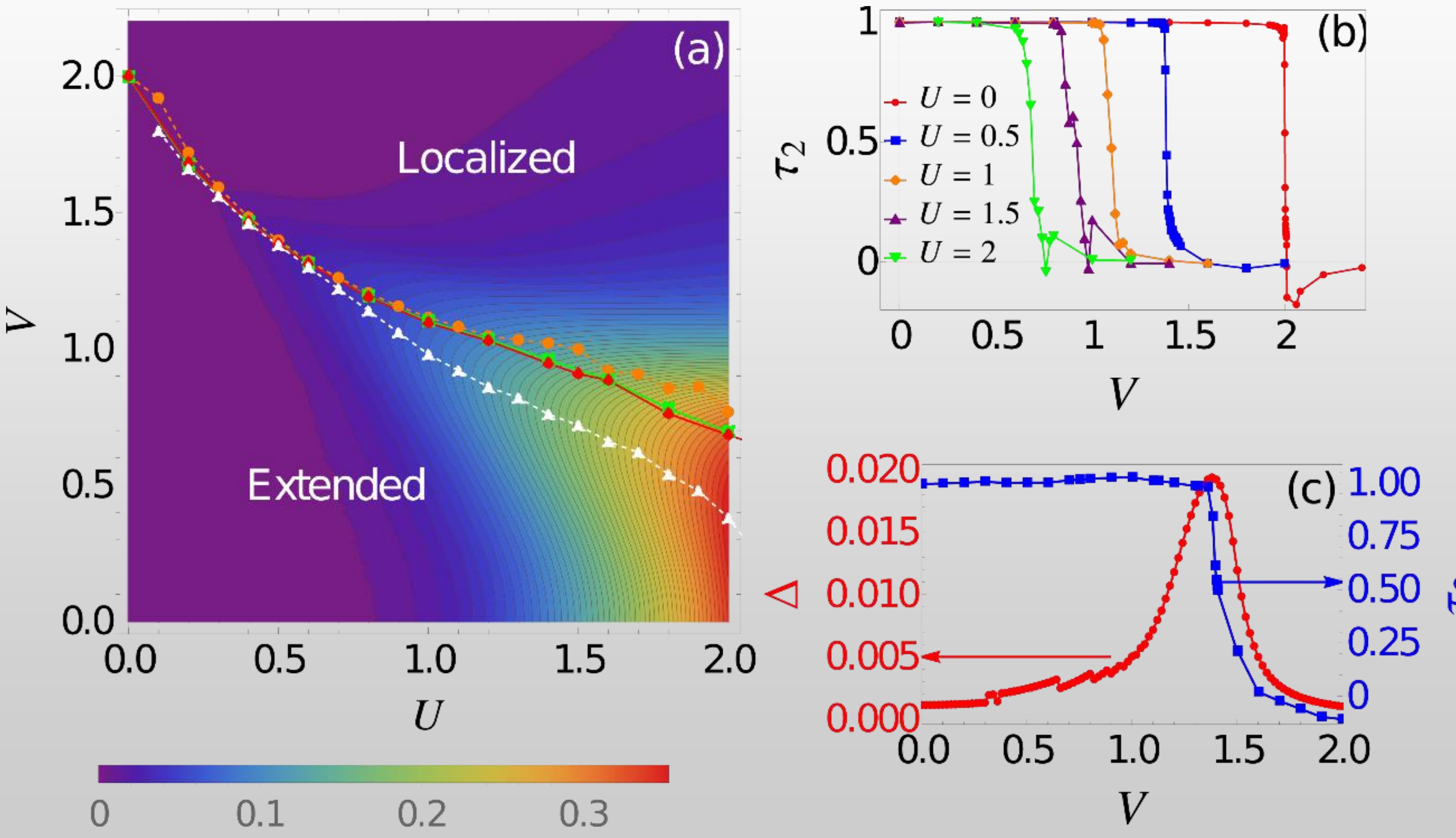
$$H = -t \sum_{i\sigma} \left(c^\dagger_{i\sigma} c_{i+1\sigma} + \mathrm{H.c.} \right) + \sum_i \left[V \cos\left(2\pi \beta_p i \right) - \mu \right] n_i - U \sum_i n_{i\uparrow} n_{i\downarrow} \,,$$

  - **Same method as Ghosal, Randeria, Trivedi (1998)**
  - **Incorporates the Hartree shift (Altshuler-Aronov corrections)**
  - **Also computed BCS superfluid stiffness**

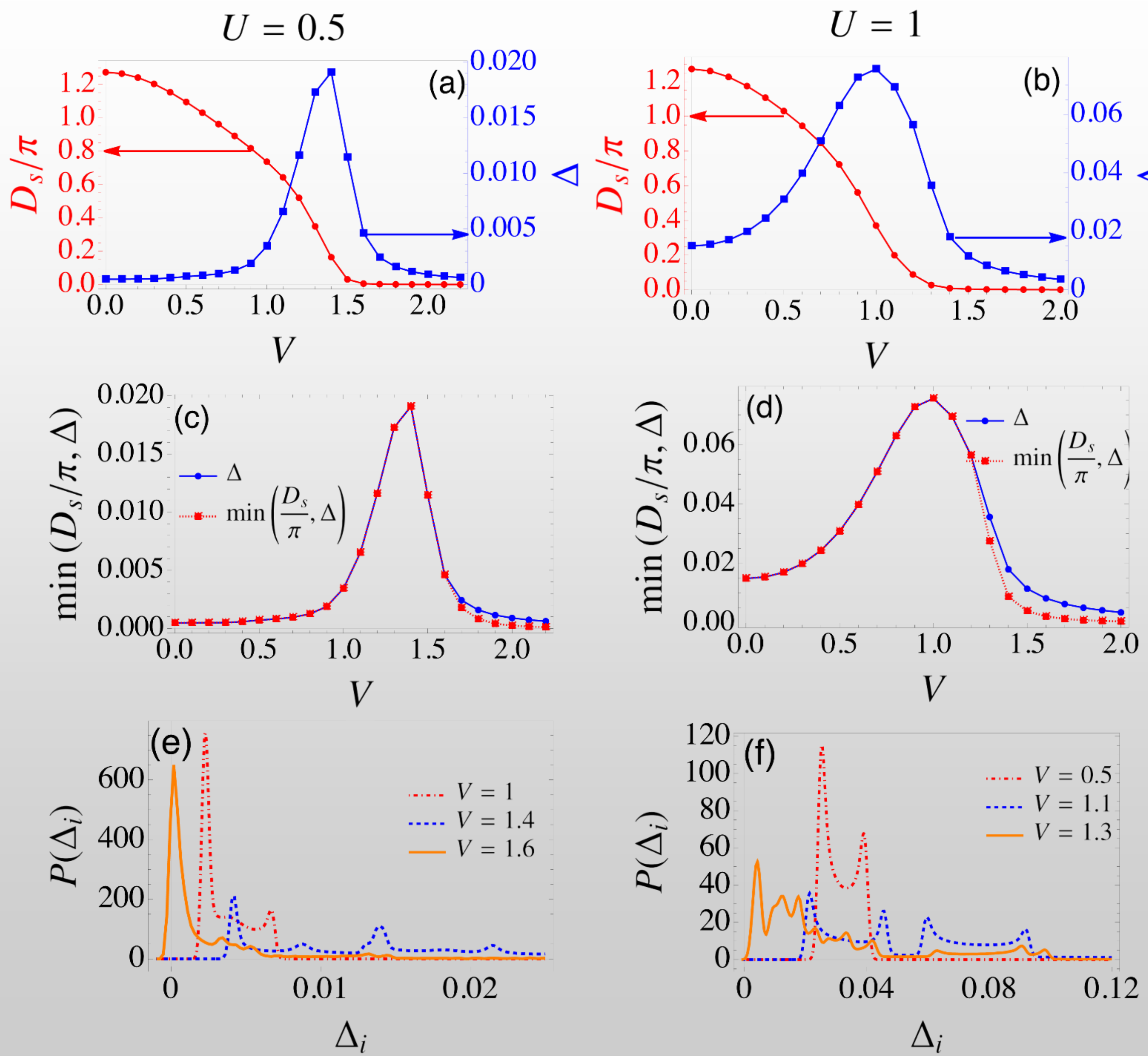
Spectrum-wide fractality *enhances superconductivity*

- **Numerics: SWQC survives at MIT**
- ***BCS pairing strongly enhanced at the transition***

Earlier evidence of enhancement:
Fan, Chern, Lin 2021

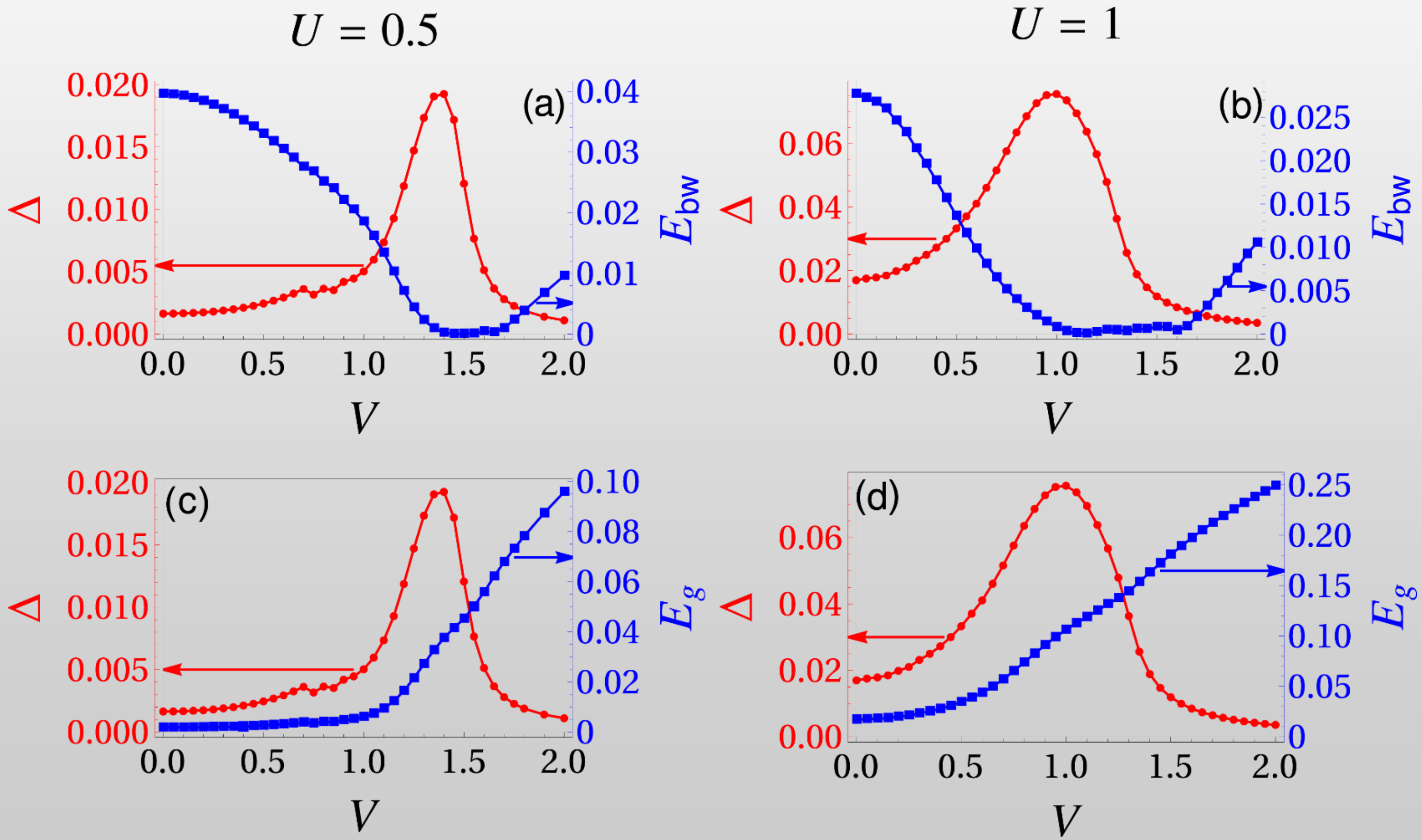


Spectrum-wide fractality *enhances superconductivity*



Spectrum-wide fractality *enhances superconductivity*

- **Interaction-dressed Hofstadter energy spectrum: DoS is enhanced near MIT (subband flattening)**
- **Plays some role in enhancement of SC**



PRBM model with attractive Hubbard

- **Alternative 1D model without DoS enhancement: Power-Law random-banded matrix model**

$$H = \sum_{i,j,\sigma} h_{i,j}\, c^\dagger_{i\sigma}\, c_{j\sigma} - U \sum_i n_{i\uparrow} n_{i\downarrow}\,, -\mu \sum_i n_i$$

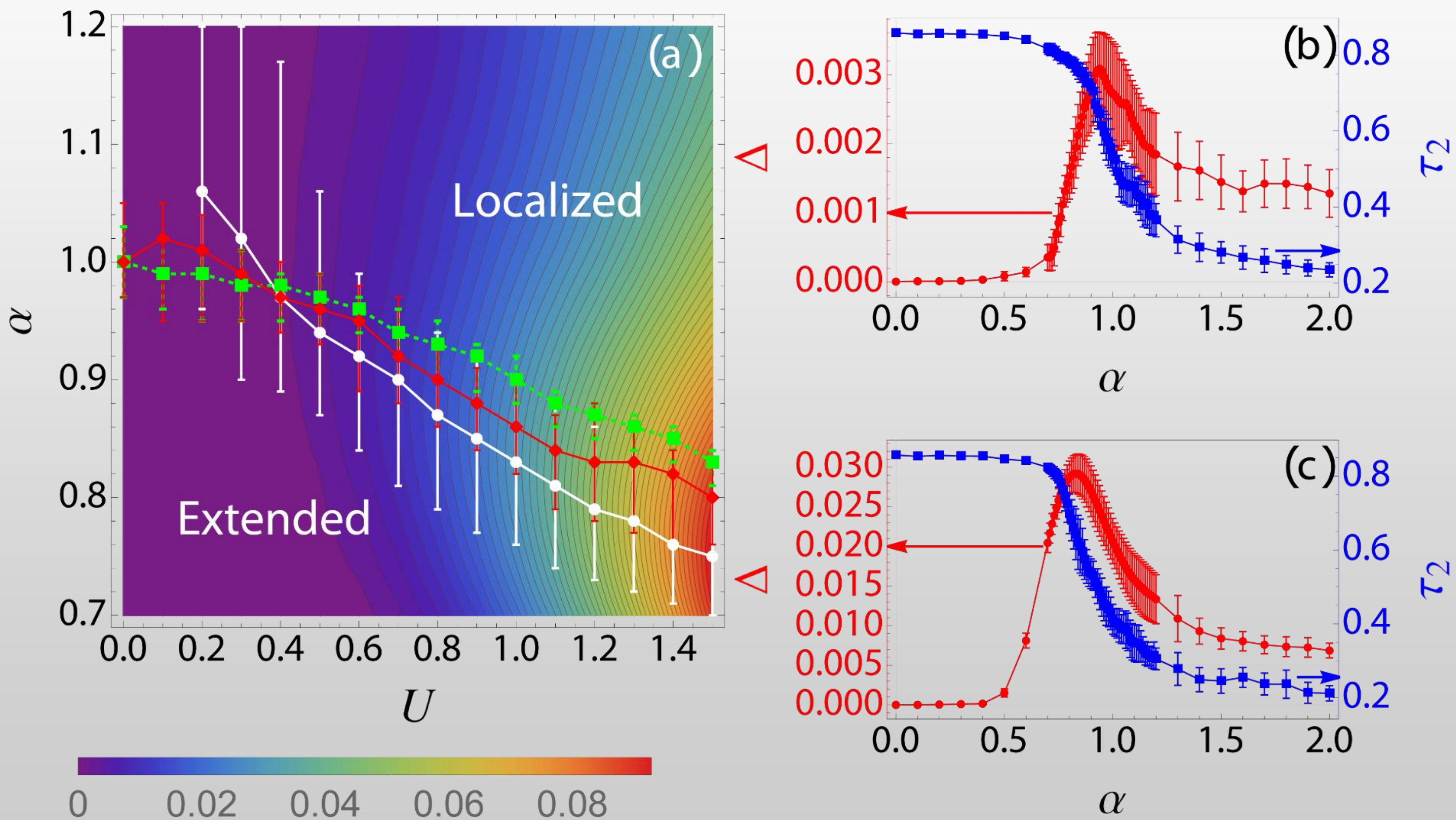
- $h_{ij} = g_{ij}\, |i-j|^{-\alpha} \quad (i \neq j),\ \ g_{ij}$ is a GOE matrix

- Without interactions:
  - $0 \leq \alpha \leq \frac{1}{2}$ Random-matrix regime
  - $\frac{1}{2} < \alpha < 1$ Ergodic (but superballistic) metallic phase
  - $\alpha = 1$ SWQC Anderson MIT
  - $\alpha > 1$ (Power-law localized) Anderson insulator

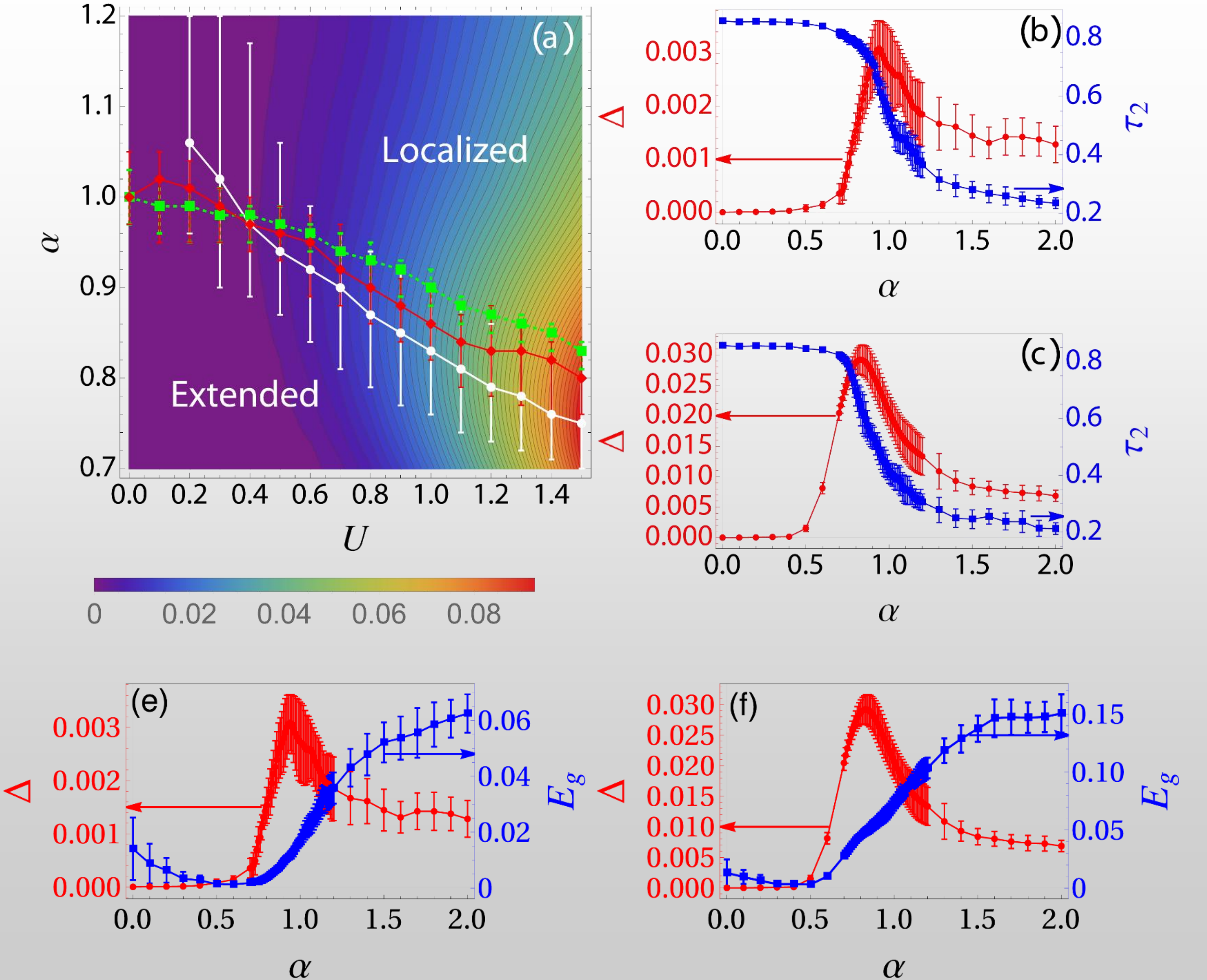
Review:
Evers and Mirlin RMP 2008

Spectrum-wide fractality *enhances superconductivity*

- **Numerics: SWQC survives at MIT**
- ***BCS pairing strongly enhanced at the transition***



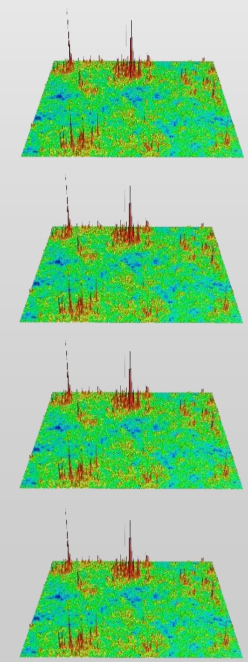
Spectrum-wide fractality *enhances superconductivity*



3D topological superconductors: Spectrum-wide quantum criticality

Class	T	P	S	Spin sym.	$d=2$	$d=3$	Topological realization	Replicated fermion NLσM
C	0	-1	0	SU(2)	$2\mathbb{Z}$	$\cdots$	SQHE (2D $d+id$ TSC)	$\mathrm{Sp}(4n)/\mathrm{U}(2n)$
A (unitary)	0	0	0	U(1)	$\mathbb{Z}$	$\cdots$	IQHE	$\mathrm{U}(2n)/\mathrm{U}(n)\otimes\mathrm{U}(n)$
D	0	$+1$	0	$\cdots$	$\mathbb{Z}$	$\cdots$	TQHE (2D $p+ip$ TSC)	$\mathrm{O}(2n)/\mathrm{U}(n)$
CI	$+1$	-1	1	SU(2)	$\cdots$	$2\mathbb{Z}$	3D TSC	$\mathrm{Sp}(4n)\otimes\mathrm{Sp}(4n)/\mathrm{Sp}(4n)$
AIII	0	0	1	U(1)	$\cdots$	$\mathbb{Z}$	3D TSC, chiral TI	$\mathrm{U}(2n)\otimes\mathrm{U}(2n)/\mathrm{U}(2n)$
DIII	-1	$+1$	1	$\cdots$	$\mathbb{Z}_2$	$\mathbb{Z}$	3D TSC (^{3}He-B)	$\mathrm{O}(2n)\otimes\mathrm{O}(2n)/\mathrm{O}(2n)$
AI (orthogonal)	$+1$	0	0	SU(2)	$\cdots$	$\cdots$	$\cdots$	$\mathrm{Sp}(4n)/\mathrm{Sp}(2n)\otimes\mathrm{Sp}(2n)$
AII (symplectic)	-1	0	0	$\cdots$	$\mathbb{Z}_2$	$\mathbb{Z}_2$	2D, 3D TIs	$\mathrm{O}(2n)/\mathrm{O}(n)\otimes\mathrm{O}(n)$
BDI	$+1$	$+1$	1	SU(2)	$\cdots$	$\cdots$	$\cdots$	$\mathrm{U}(2n)/\mathrm{Sp}(2n)$
CII	-1	-1	1	$\cdots$	$\cdots$	$\mathbb{Z}_2$	3D chiral TI	$\mathrm{U}(2n)/\mathrm{O}(2n)$

- Replica symmetry suggests that 2D surface states of bulk TSCs can ***avoid Anderson localization*** via a strange mechanism that connects to topological quantum phase transitions in 2D
- "Spectrum-wide quantum criticality" (SWQC): ***all single-particle wave functions are quantum critical***, i.e. are neither localized nor extended, but random and critically rarified, with a universal spectrum of level-set statistics ("multifractality")
- SWQC for 2D surface states of 3D TSCs: **All finite-energy surface states are 2D quantum-Hall plateau-transition states!!**



Spectrum-wide quantum criticality

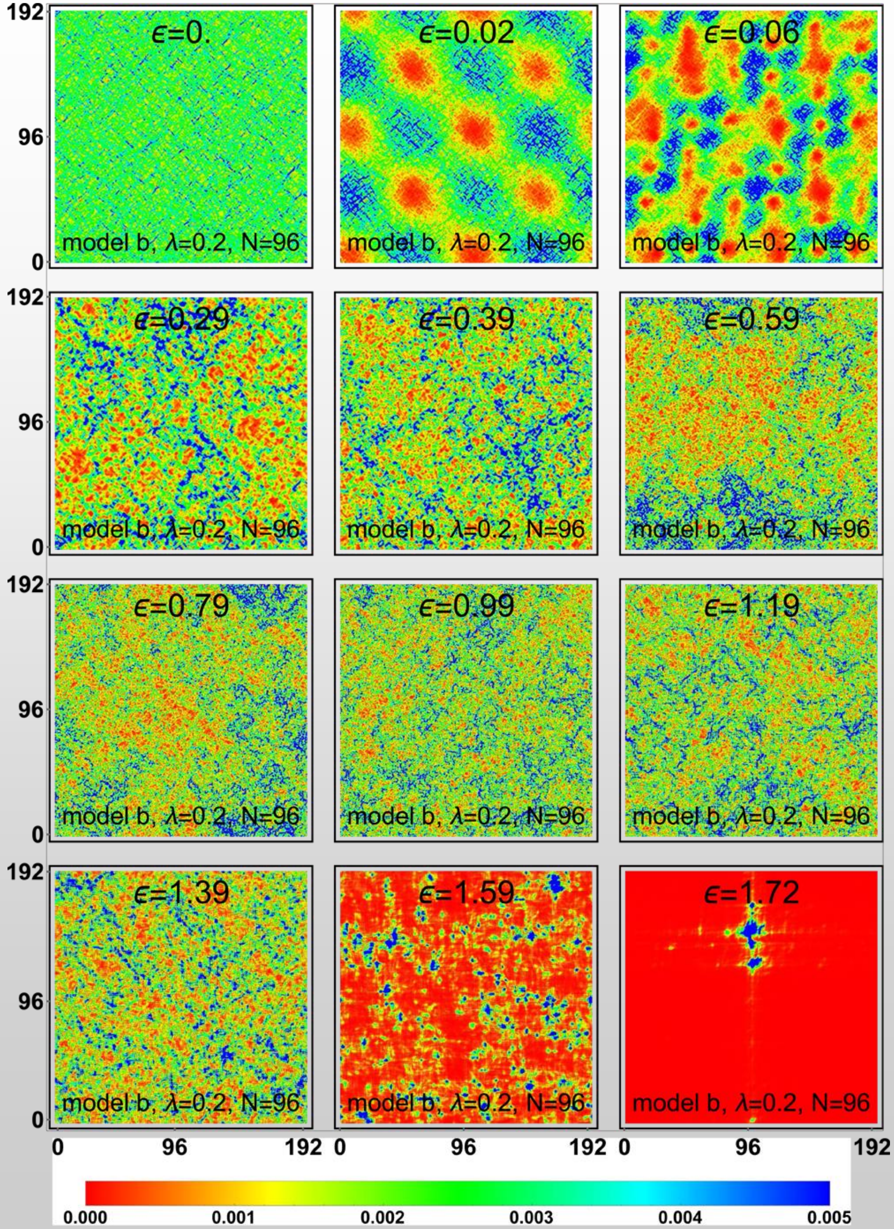
Surface state of the simplest, class DIII topological superconductor ("solid-state Helium 3B"):

- **Single, 2-component massless Majorana fermion**
- **No conserved charge (e.g. spin) to gauge!**
- **Only energy is conserved**
- **Gauging energy by coupling to the stress tensor: Quenched gravity! (formally irrelevant at zero energy)**

$$H^{(1)}_{\mathsf{DIII}} = -\frac{1}{2} \sum_{a,b=1,2} \int d^2\mathbf{r}\, v_{ab}(\mathbf{r}) \left(\bar{\psi} i \hat{\sigma}^a \overleftrightarrow{\partial}_b \psi \right)$$

Ghorashi, Karcher, Davis, Foster 2020

Spectrum-wide quantum criticality



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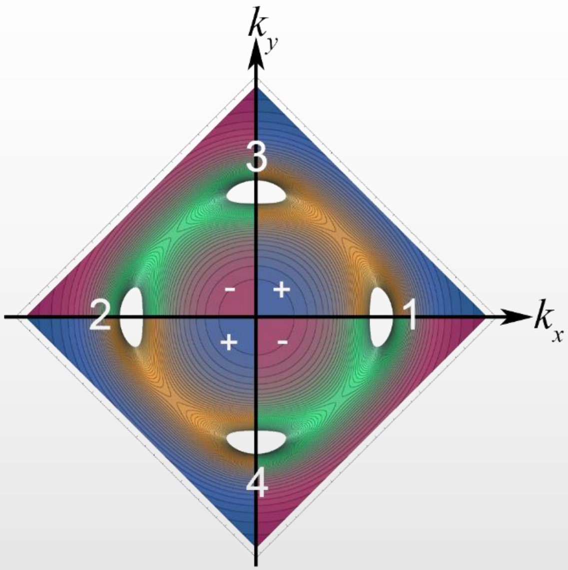
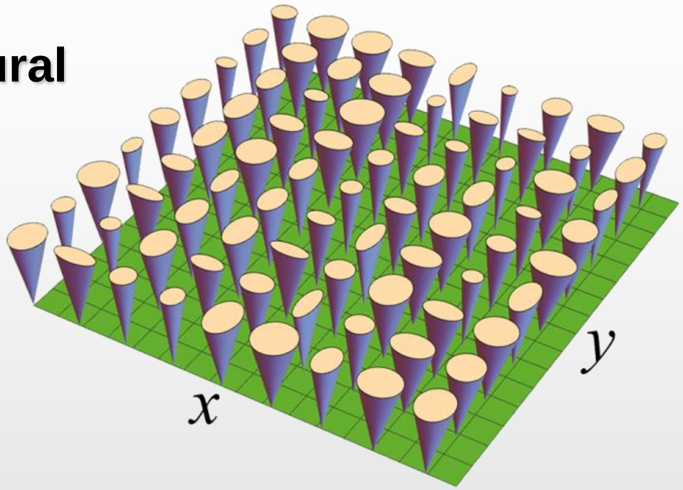
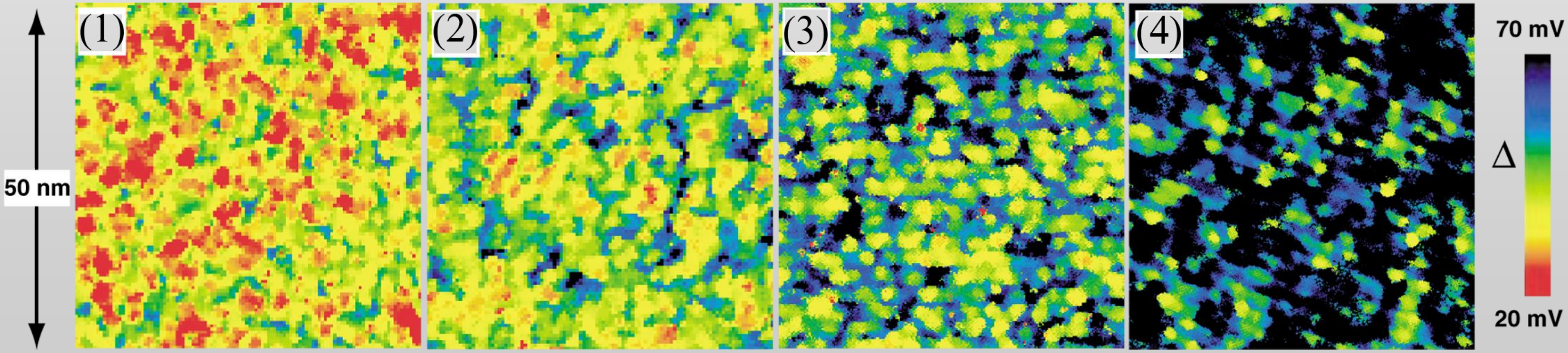
Ghorashi, Karcher, Davis, Foster 2020

Beyond topology: *Dirty d-wave quasiparticles redux*



Nematic disorder: natural version of quenched gravitational dirt





BSCCO STM Data: K. McElroy, J. C. Davis *et al.*, PRL (2005)

Summary: Some stuff since covid-19

1. Shock + Bounce PRL 127, 026801 (2021)
2. Fractal SC PRB 106, L180503 (2022)
3. Magnetism at the onset of a spectrum-wide quantum-critical transition (in preparation)

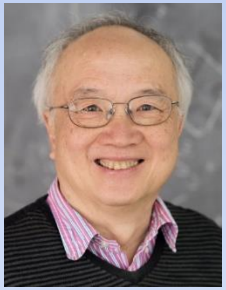
Xinghai Zhang
Rice University

Tsz Chun Wu

Rice University,
Trexquant

Yunxiang Liao

JQI/CMTC
U. Maryland,
KTH



Patrick Lee

MIT

1. Quantum interference in a dirty MFL PRB 106, 155108 (2022)
2. Enhancement of SC in a dirty MFL arXiv:2305.13357 (2023)
3. Spectrum-wide quantum criticality:
   Review Karcher and Foster Ann. Phys. 435, 168439 (2021)

Jonas Karcher
Penn State