Interaction-mediated surface state instability in dirty topological superconductors

Matthew S. Foster[1] and Emil A. Yuzbashyan[2]

[1] Rice University, [2] Rutgers University

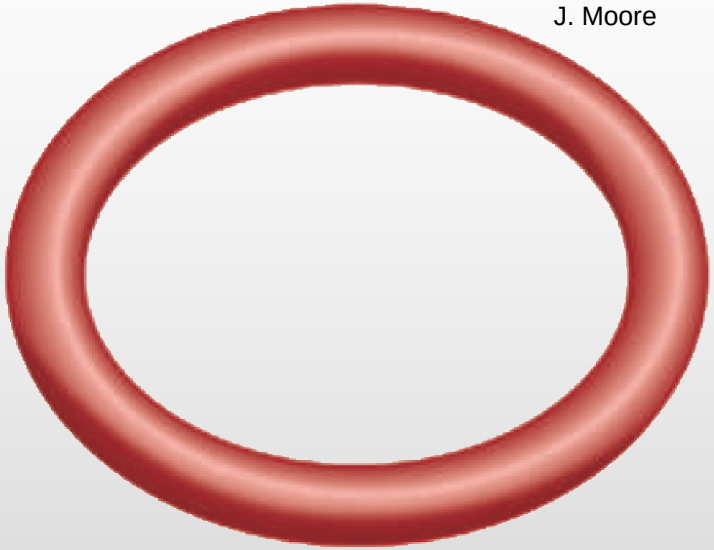
May 3rd, 2013

PRL 109, 246801 (2012); in preparation

Topological insulators: A knotty state of affairs

Topologically non-trivial bulk

"Unwinding" edge/surface state



J. Moore

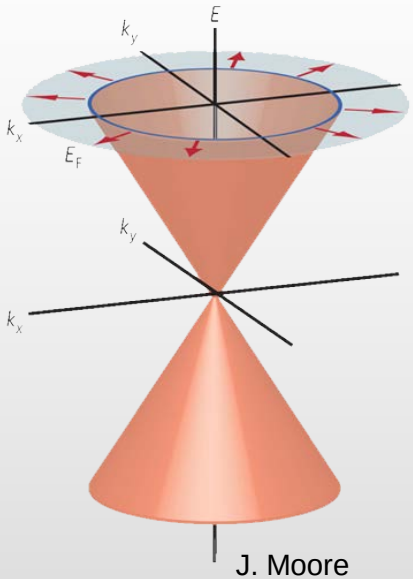
Topologically trivial vacuum

3D topological insulator theory:

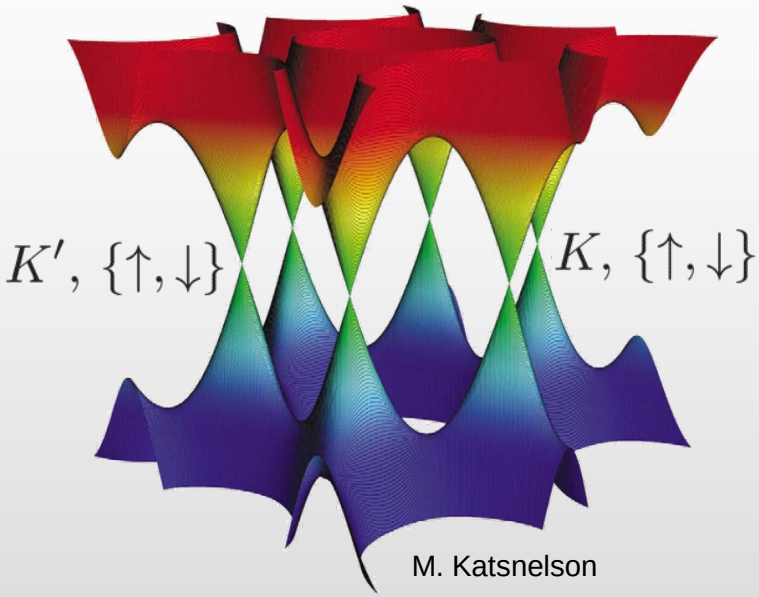
- Fu, Kane, and Mele 2007
- Moore and Balents 2007
- Qi, Hughes, and Zhang 2008
- R. Roy 2009

The "Smoking gun": *Single* Dirac surface state on Bi_2Se_3 and Bi_2Te_3

Topological surface state on Bi_2Se_3 = "¼ of graphene"



J. Moore



M. Katsnelson

- **"Holography"**
  ***Unpaired* 2D Dirac fermion in Time-Reversal Invariant system only allowed as surface of 3D bulk**
- **Half-integer Quantum Hall effect (parity anomaly)**
- **Quantized magnetoelectric coupling, axion electrodynamics…**

Fu, Kane, and Mele 2007, Moore and Balents 2007, Fu and Kane 2007; Qi, Hughes, and Zhang 2008, R. Roy 2009

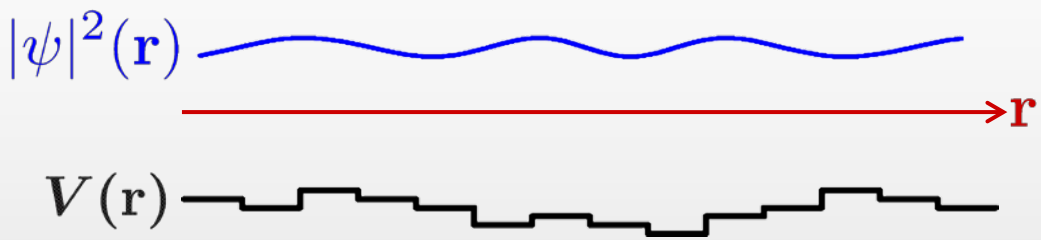
Anderson Localization: Basics

Philip Anderson, 1958:

A sufficiently strong random potential $V(\mathbf{r})$ exponentially localizes all single particle states (at a given energy)

Weak disorder:

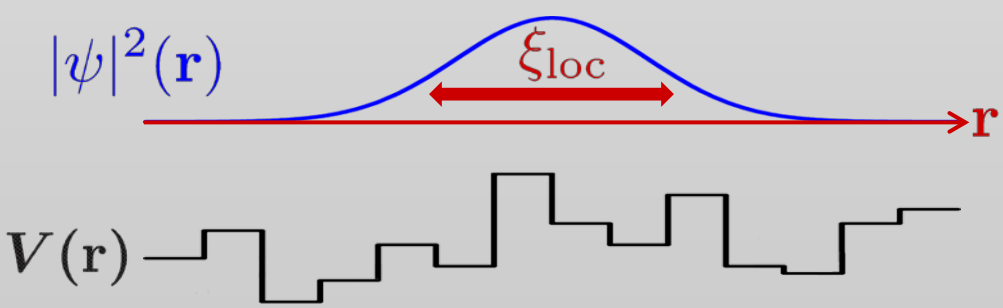
Extended states



Strong disorder:

Localized states

Characterized by localization length ξ_{loc}



Anderson Localization: Basics

Philip Anderson, 1958:

A sufficiently strong random potential $V(\mathbf{r})$ exponentially localizes all single particle states (at a given energy)

Weak disorder:

Extended states

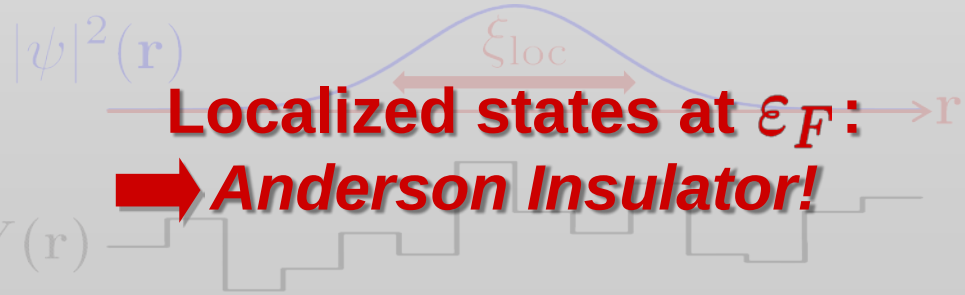
Extended states at ε_F:

➡ ***(Dirty) Metal!***

Strong disorder:

Localized states

Characterized by localization length ξ_{loc}



Localized states at ε_F:

➡ ***Anderson Insulator!***

The multifractal spectrum $\tau(q)$: Probe for quantum interference

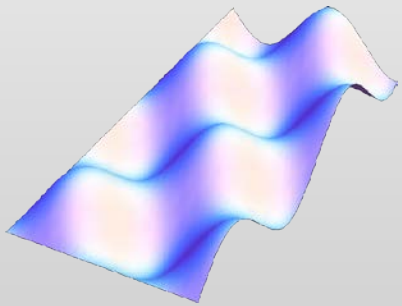
- **"Topologically-protected surface states"**
  Protection from Anderson localization!

Bardarson, Tworzydlo, Brouwer, Beenakker 2007
Nomura, Koshino, Ryu 2007
Ryu, Mudry, Obuse, Furusaki 2007
Ostrovsky, Gornyi, Mirlin 2007

- **Localized or not? Inverse Participation Ratio (IPR)**

$$P_q \equiv \int_{L^d} |\psi(\mathbf{r})|^{2q} \, d^d\mathbf{r} \sim \left(\frac{a}{L}\right)^{\tau(q)}$$

- **As a probe of wavefunction (de)localization:**

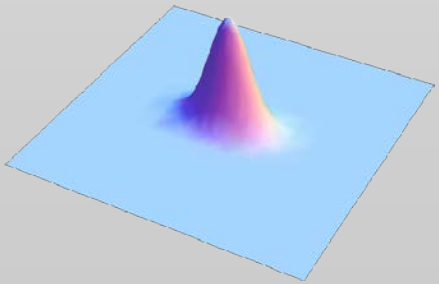


(a) Extended plane wave state

$$\tau(q) = d(q-1)$$

(b) Exponentially localized state

$$\tau(q) \sim 0, \quad L \gg \xi_{\mathrm{loc}}$$



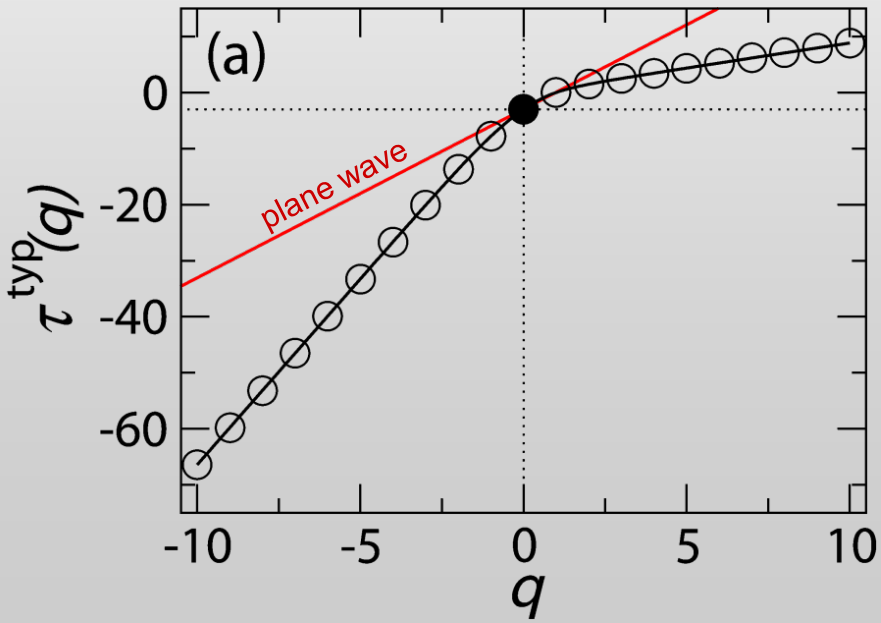
Multifractal spectrum: "Fingerprint" of the MIT

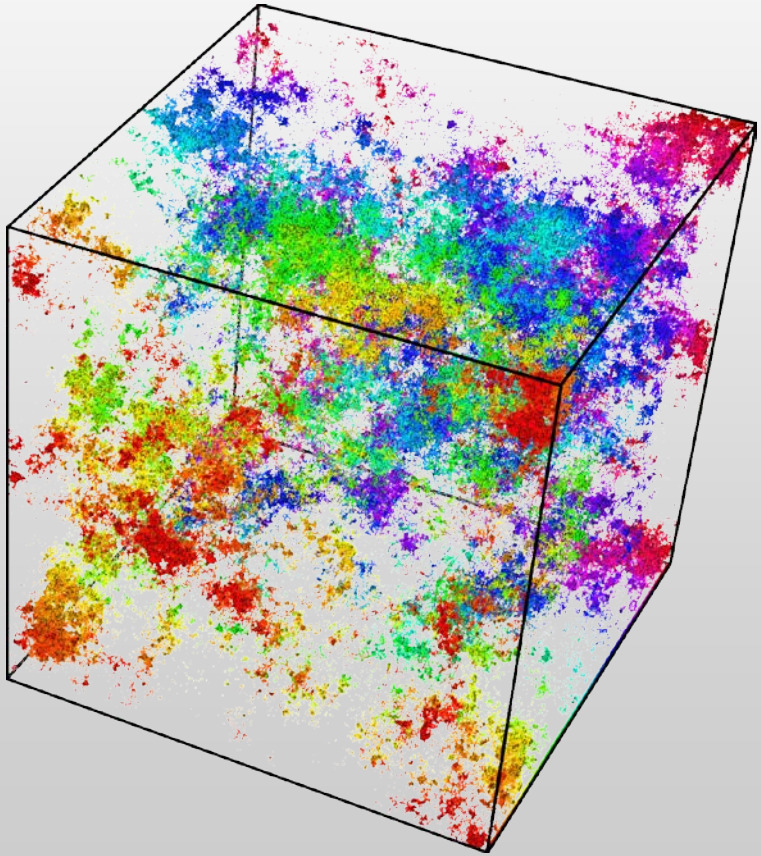
Critical wavefunctions at a delocalization transition

Example: Anderson Metal to Insulator transition in 3D

- ψ **exhibits *multifractal scaling***
  (neither localized nor extended, nor a simple fractal)
  Wegner 1980; Castellani and Peliti 1986

- **Multifractality encoded in *nonlinearity* of the $\tau(q)$ spectrum**





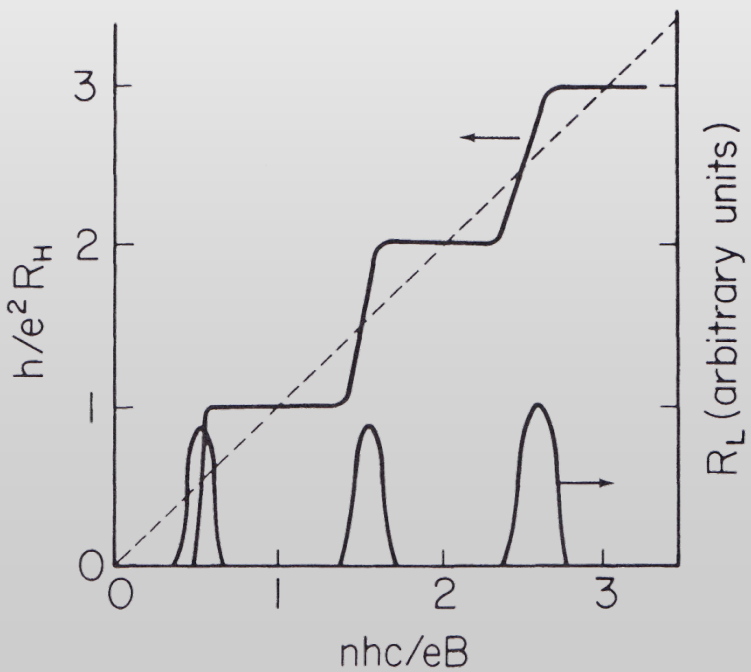
Vasquez, Rodriguez, Roemer 2008

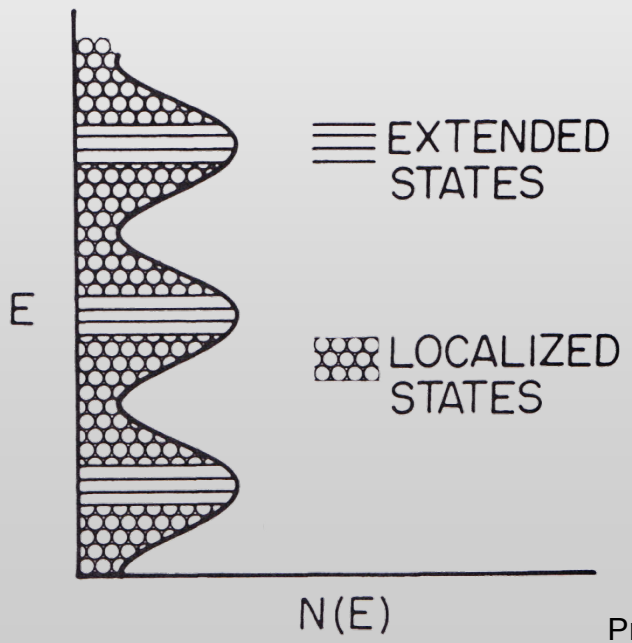
Multifractal spectrum: *Universal Fingerprint for quantum criticality*

Wavefunction multifractality at a delocalization transition

Example: Multifractal spectra at the Integer Quantum Hall Plateau Transition

- States away from LL center are Anderson localized
- *Delocalized states* at the transition (non-zero longitudinal resistance/conductance)





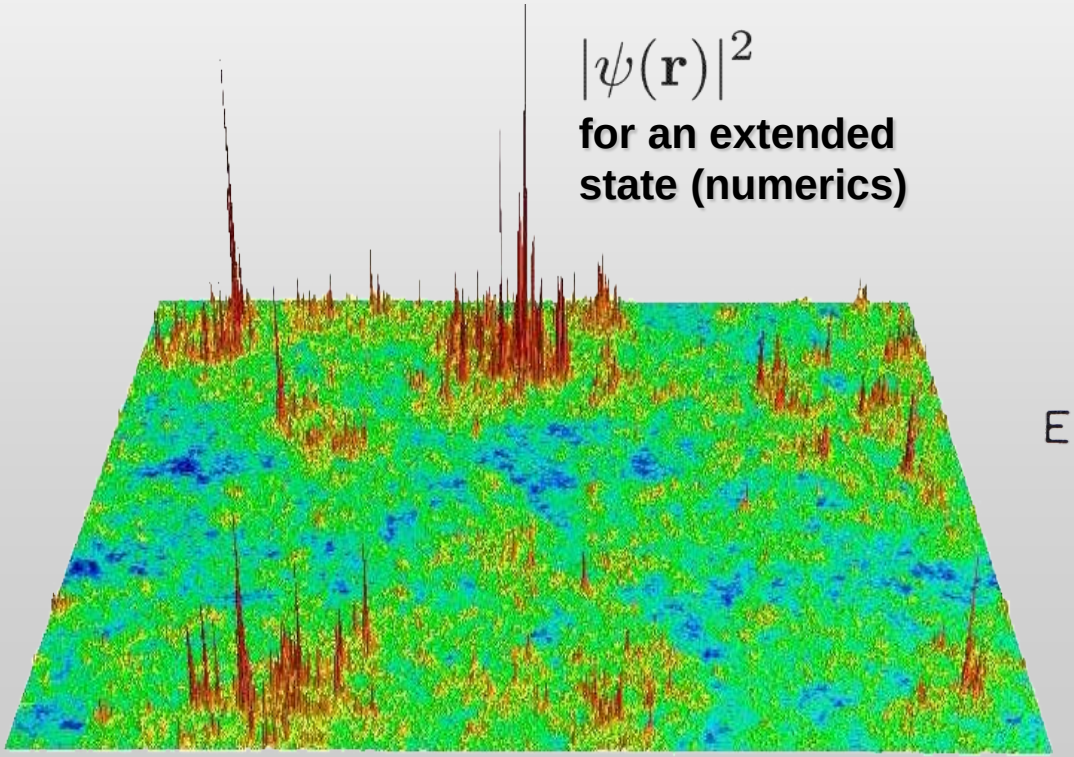
Prange 1987

Multifractal spectrum: *Universal Fingerprint for quantum criticality*

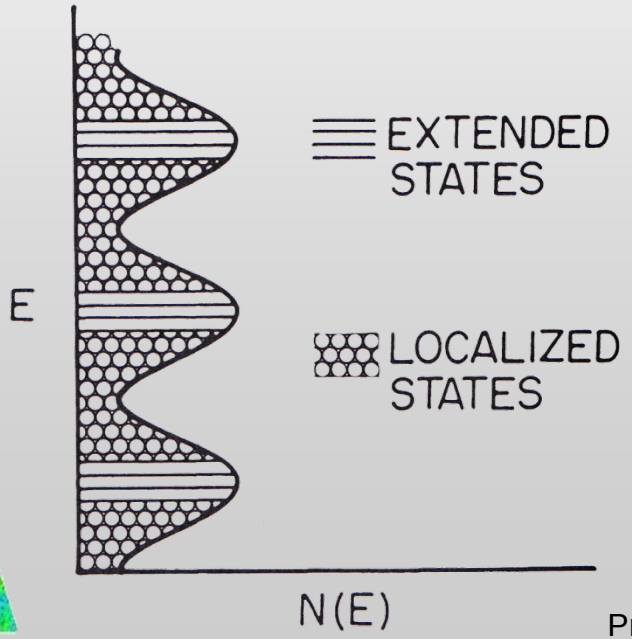
Wavefunction multifractality at a delocalization transition

Example: Multifractal spectra at the Integer Quantum Hall Plateau Transition

- States away from LL center are Anderson localized
- *Delocalized states* at the transition (non-zero longitudinal resistance/conductance)



F. Evers



Prange 1987

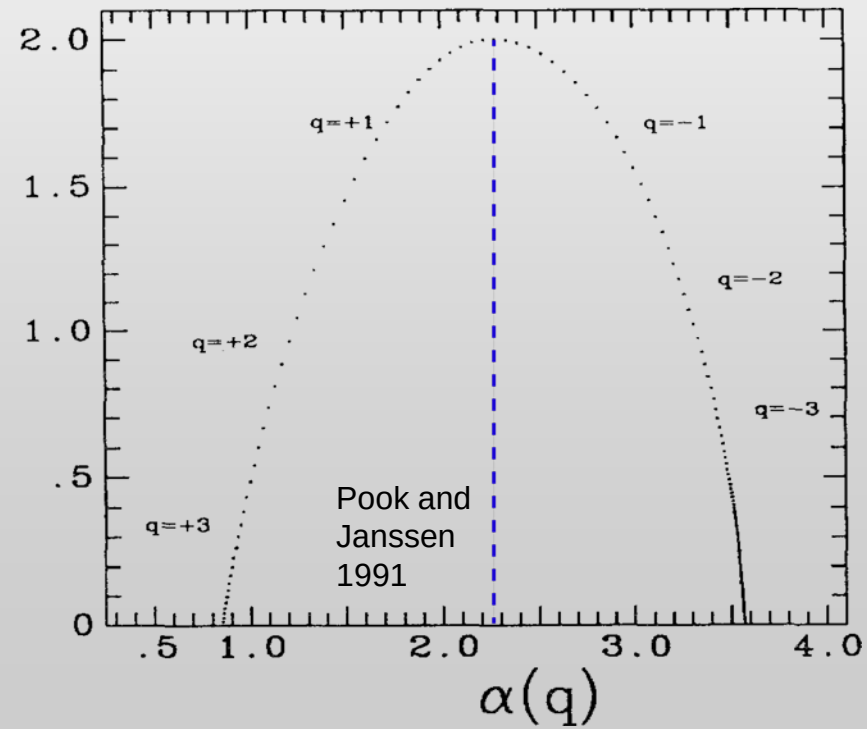
Multifractal spectrum: *Universal Fingerprint for quantum criticality*

Wavefunction multifractality: plateau transition

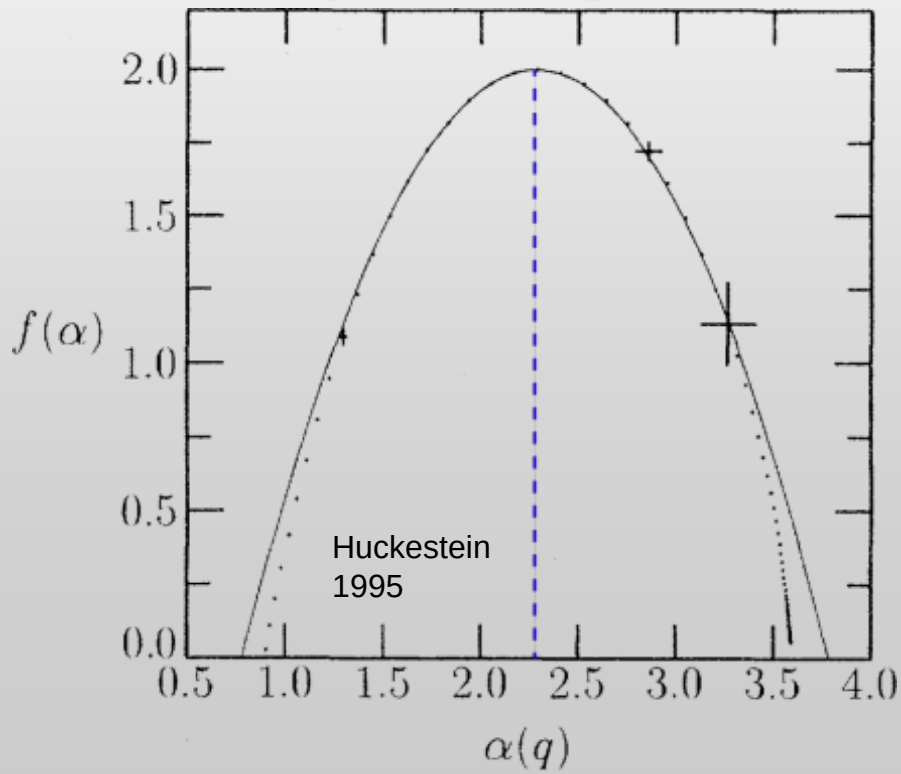
- $\tau(q)$ is both <u>self-averaging</u> and <u>universal</u>
- $f(\alpha) \equiv \alpha q - \tau(q)$ **"singularity spectrum"**

Chamon, Mudry, Wen 1996;
Mirlin and Evers 2000;
Obuse, Subramaniam, Furusaki, Gruzberg, Ludwig 2008;
Evers, Mildenberger, Mirlin 2008

Real space Hamiltonian



Tight-binding model



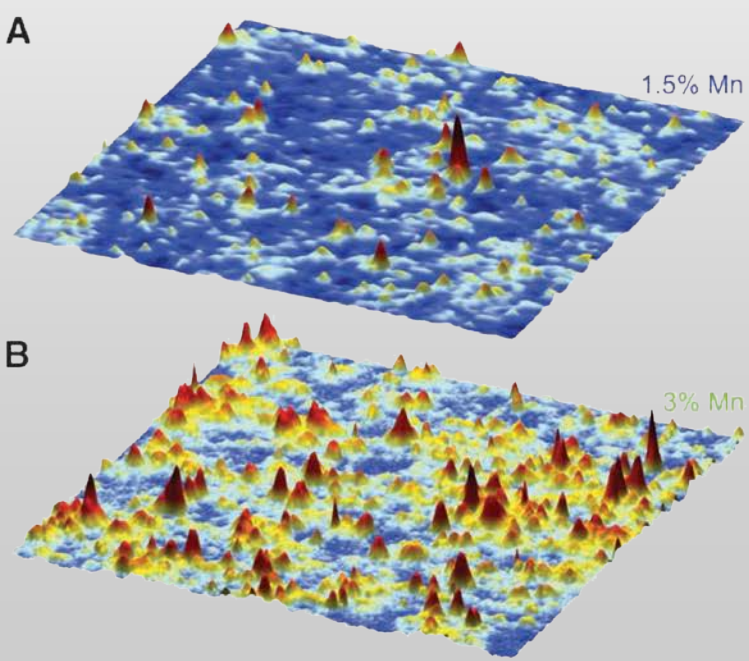
The multifractal spectrum $\tau(q)$: Probe for quantum interference

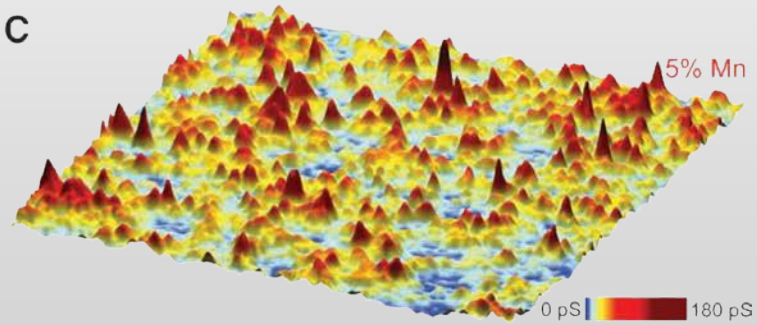
- **Experiment? On *metallic side*, can use LDOS $\rho(\varepsilon,\mathbf{r})$ instead:**

$$P_q = \frac{\int_{L^d} \rho^q(\varepsilon,\mathbf{r})\, d^d\mathbf{r}}{\left[\int_{L^d} \rho(\varepsilon,\mathbf{r})\, d^d\mathbf{r}\right]^q} \sim \left(\frac{a}{L}\right)^{\tau(q)}$$

Multifractal spectra for topological insulator surface states

Foster PRB 2012





Multifractal LDOS fluctuations in GaMnAs

Richardella, Roushan, Mack, Zhou, Huse, Awschalom, and Yazdani 2010

Topological Insulators and Superconductors: The 10-Fold Way

Schnyder, Ryu, Furusaki, Ludwig 2008, 2010; Kitaev 2009:

- **In any number *d* of spatial dimensions, 5 symmetry classes topological materials, topologically-protected surface states**
- **These are a *subset* of 10 random matrix classes (also used in Anderson localization)**

	d								
Cartan	0	1	2	3	4	5	6	7	8
Complex case:									
A	Z	0	Z	0	Z	0	Z	0	Z
AIII	0	Z	0	Z	0	Z	0	Z	0
Real case:									
AI	Z	0	0	0	2Z	0	Z_2	Z_2	Z
BDI	Z_2	Z	0	0	0	2Z	0	Z_2	Z_2
D	Z_2	Z_2	Z	0	0	0	2Z	0	Z_2
DIII	0	Z_2	Z_2	Z	0	0	0	2Z	0
AII	2 Z	0	Z_2	Z_2	Z	0	0	0	2Z
CII	0	2 Z	0	Z_2	Z_2	Z	0	0	0
C	0	0	2Z	0	Z_2	Z_2	Z	0	0
CI	0	0	0	2 Z	0	Z_2	Z_2	Z	0

Schnyder, Ryu, Furusaki, Ludwig 2010

Green:
3D Z_2 Topological Insulator (Bi_2Se_3, etc)

Red:
3D Topological Superconductors (TSC)

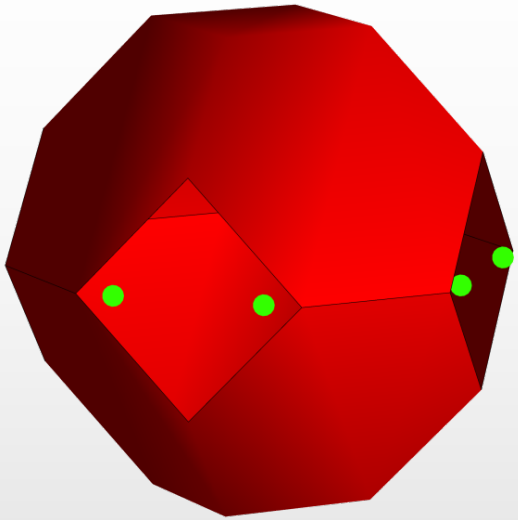
Zirnbauer 1996
Altland and Zirnbauer 1997
Bernard and LeClair 2002

Model Spin Singlet 3D Topological Superconductor

Spin singlet TSC: class CI [spin SU(2) symmetry]

3D Lattice model

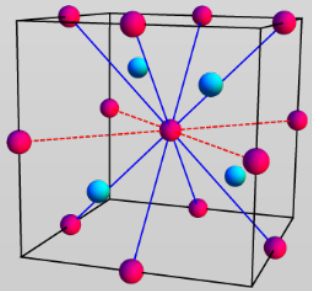
Schnyder, Ryu, Ludwig 2009



$$H_{\text{bulk}} = \sum_{i,j} \left[t_{ij} c^\dagger_{i,s} c_{j,s} + \Delta_{ij} c^\dagger_{i,\uparrow} c^\dagger_{j,\downarrow} + \text{H.c.} \right]$$

- **"Haldane-Kane-Mele" type model**
- **Spin ½ electrons**
- **No spin-orbit coupling**
- **nn hopping on the diamond lattice at half-filling**
- **nnn d-wave pairing—3D BdG quasiparticle nodes**
  Read and Green 2000:
  non-trivial topology ⟷ non-s wave pairing

d-wave
nnn pairing



Bulk not fully gapped!

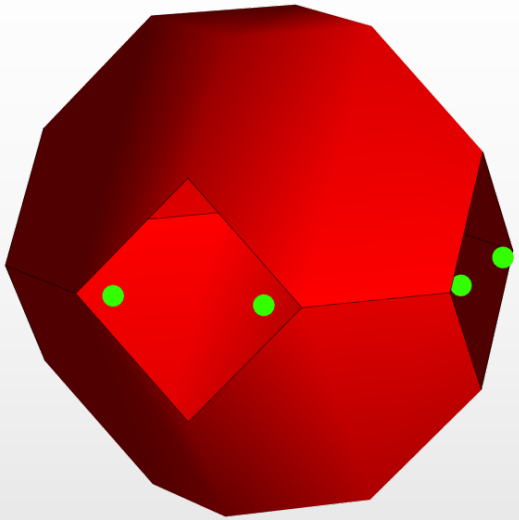
Not yet a topological superconductor!

Model Spin Singlet 3D Topological Superconductor

Spin singlet TSC: class CI [spin SU(2) symmetry]

3D Lattice model

Schnyder, Ryu, Ludwig 2009



- ***Bulk* quasiparticle mass gap:**

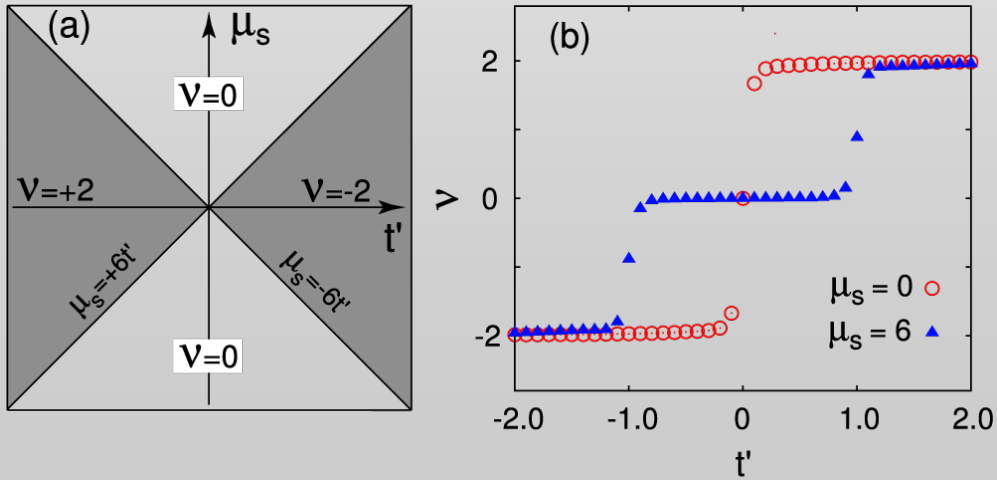
  (a) sublattice staggered chemical potential μ_s (trivial)

  (b) nnn hopping t' ("topological")

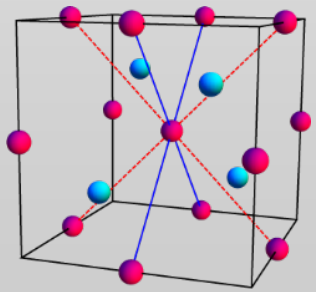
- **Winding number:**

$$\nu[q] \equiv \int \frac{d^3\mathbf{k}}{24\pi^2} \epsilon^{\mu\nu\rho} \, \mathrm{Tr}\left[(q^{-1}\partial_\mu q)(q^{-1}\partial_\nu q)(q^{-1}\partial_\rho q)\right]$$
$$\in \{-2, 0, 2\}$$

Schnyder, Ryu, Furusaki, Ludwig 2008



nnn hopping *t'*



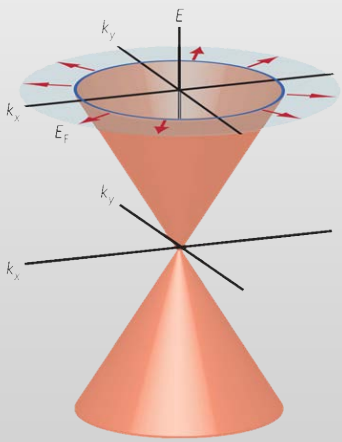
Topological superconductor surface states

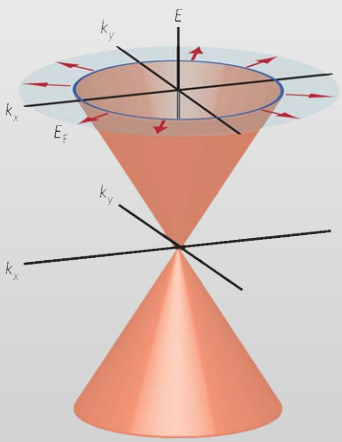
Spin singlet TSC: class CI [spin SU(2) symmetry]

- **Topologically-protected, gapless surface state (Bogoliubov) quasiparticles**
- **$|\nu| = 2k$ valleys, $k = (1,2,3,\ldots)$; SRL model has $k = 1$**

Low energy surface Andreev state Hamiltonian:

$$H = \int d^2\mathbf{r}\, \Psi^\dagger \left(-i\hat{\boldsymbol{\sigma}} \cdot \boldsymbol{\nabla}\right) \Psi$$





- **Surface "Majorana" (Andreev bound state) band fermion** $\Psi_v = \begin{bmatrix} c_{\uparrow,v} \\ c^\dagger_{\downarrow,v} \end{bmatrix}$

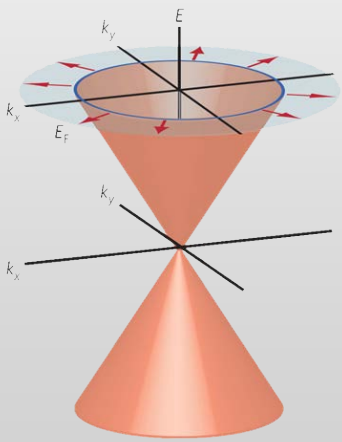
Topological superconductor surface states

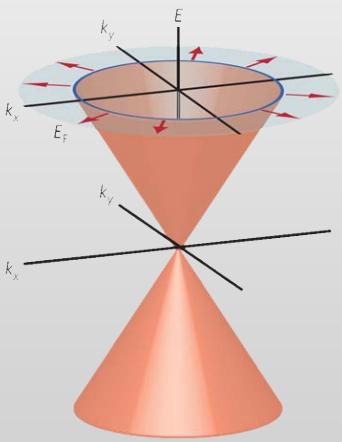
Spin singlet TSC: class CI [spin SU(2) symmetry]

- **Topologically-protected, gapless surface state (Bogoliubov) quasiparticles**
- **$|\nu| = 2k$ valleys, $k = (1,2,3,\ldots)$; SRL model has $k = 1$**

Low energy surface Andreev state Hamiltonian:

$$H = \int d^2\mathbf{r}\, \Psi^\dagger \left(-i\hat{\boldsymbol{\sigma}} \cdot \boldsymbol{\nabla}\right) \Psi \equiv \Psi^\dagger \hat{h} \Psi$$





- **"Topological" chiral symmetry (= *physical time-reversal*):** $-\hat{\sigma}^3 \hat{h} \hat{\sigma}^3 = \hat{h}$
- **"Particle-hole" symmetry [= *spin SU(2) invariance*]:** $-\hat{\sigma}^1 \hat{\kappa}^2 \hat{h}^\mathsf{T} \hat{\sigma}^1 \hat{\kappa}^2 = \hat{h}$

Topological superconductor surface states

Spin singlet TSC: class CI [spin SU(2) symmetry]

- **Topologically-protected, gapless surface state (Bogoliubov) quasiparticles**
- **$|\nu| = 2k$ valleys, $k = (1,2,3,\ldots)$; SRL model has $k = 1$; *n* replicas**

Low energy surface Andreev state Hamiltonian:

$$H = \int d^2\mathbf{r}\, \Psi^\dagger \left(-i\hat{\boldsymbol{\sigma}} \cdot \boldsymbol{\nabla}\right) \Psi$$

$$= -\int d^2\mathbf{r}\, \left(\mathsf{L}\, \bar{\partial}\mathcal{L} + \mathsf{R}\, \partial\mathcal{R}\right)$$

$$\mathcal{L} \equiv \begin{bmatrix} \Psi_1 \\ -\hat{\kappa}^2 \left(\Psi_2^\dagger\right)^\mathsf{T} \end{bmatrix} \rightarrow \mathcal{L}_{s,v,a}$$

spin
valley
replica

$$\mathsf{L} \equiv \mathcal{L}^\mathsf{T} i\hat{s}^2 \hat{\kappa}^2$$

Invariant under chiral (independent left and right) transformations:

- **Spin SU(2) rotations** $\quad \mathcal{L}_{s,v,a} \rightarrow \left[\exp(-i\hat{\vec{S}} \cdot \vec{\phi})\right]_{s,s'} \mathcal{L}_{s',v,a}$
- **Valley Sp(2 k) rotations** $\quad \mathcal{L}_{s,v,a} \rightarrow \left[\exp(-i\hat{t}^\eta_\kappa\, \theta^\eta)\right]_{v,v'} \mathcal{L}_{s,v',a}$
- **SO(n) replica space rotations** $\quad \mathcal{L}_{s,v,a} \rightarrow \left[\exp(-i\hat{T}^\gamma_R\, \xi^\gamma)\right]_{a,a'} \mathcal{L}_{s,v,a'}$

Topological superconductor surface states

Spin singlet TSC: class CI [spin SU(2) symmetry]

- **Topologically-protected, gapless surface state (Bogoliubov) quasiparticles**
- **|ν| = 2*k* valleys, *k* = (1,2,3,…); SRL model has *k* = 1; *n* replicas**

Conformal Field Theory: SO(4 *n k*)$_1$ (free fermions)

$$H = \int d^2\mathbf{r}\, \Psi^\dagger \left(-i\hat{\boldsymbol{\sigma}} \cdot \boldsymbol{\nabla}\right) \Psi$$

$$= -\int d^2\mathbf{r}\, \left(\mathsf{L}\, \bar{\partial}\mathcal{L} + \mathsf{R}\, \partial\mathcal{R}\right)$$

$$\mathcal{L} \equiv \begin{bmatrix} \Psi_1 \\ -\hat{\kappa}^2 \left(\Psi_2^\dagger\right)^\mathsf{T} \end{bmatrix} \rightarrow \mathcal{L}_{s,v,a}$$

spin
valley
replica

$$\mathsf{L} \equiv \mathcal{L}^\mathsf{T} i\hat{s}^2 \hat{\kappa}^2$$

Invariant under chiral (independent left and right) transformations:

- ***Combined spin x valley x replica* SO(4 n k) *rotations***

$$\mathcal{L} \rightarrow \exp\left(-i\hat{S}^\Sigma\, \hat{t}^i_\kappa\, \hat{T}^\gamma_R\, \Xi^{\Sigma i \gamma}\right) \mathcal{L}$$

Multifractal topological superconductor surface states

Spin singlet TSC: class CI [spin SU(2) symmetry]

- **Topologically-protected, gapless surface state (Bogoliubov) quasiparticles**
- **TRI disorder ➡ multifractal surface states (*n* replicas, 2 *k* valleys)**

Conformal Field Theory: SO(4 *n k*)$_1$ ⊃ Sp(2 *n*)$_k$ ⊕ Sp(2 *k*)$_n$

$$H = \int d^2\mathbf{r}\, \Psi^\dagger \left(-i\hat{\boldsymbol{\sigma}}\cdot\boldsymbol{\nabla} + \mathbf{A}_i\cdot\boldsymbol{\sigma}\,\hat{t}^i_\kappa\right)\Psi = H_0 + \int d^2\mathbf{r}\, J^i_\kappa\,\bar{A}_i + \bar{J}^i_\kappa\,A_i$$

- **"Topological" chiral symmetry (= *physical time-reversal*):** $-\hat{\sigma}^3\hat{h}\hat{\sigma}^3 = \hat{h}$
- **"Particle-hole" symmetry [= *spin Sp(2) invariance*]:** $-\hat{\sigma}^1\hat{\kappa}^2\hat{h}^{\mathsf{T}}\hat{\sigma}^1\hat{\kappa}^2 = \hat{h}$

Multifractal topological superconductor surface states

Spin singlet TSC: class CI [spin SU(2) symmetry]

- **Topologically-protected, gapless surface state (Bogoliubov) quasiparticles**
- **TRI disorder ➡ multifractal surface states (*n* replicas, 2 *k* valleys)**

Conformal Field Theory: $SO(4\,n\,k)_1 \supset Sp(2\,n)_k \oplus Sp(2\,k)_n$

- LDOS multifractal spectrum [$Sp(2\,n)_k$ primary fields]

$$\tilde{\tau}(q) = 2(q-1) + \Delta_q - q\Delta_1 = (q-1)\left[2 - \frac{1}{2(k+1)}q\right]$$

***k = 1*:**
Nersesyan, Tsvelik, Wenger 1994
Mudry, Chamon, Wen 1996
Caux, Kogan, Tsvelik 1996

***k > 1*:**
Foster and Yuzbashyan 2012

- **Extended, multifractal surface states:**
  No Anderson localization = topological protection!

Bernard and LeClair 2002
Schnyder, Ryu,
Furusaki, Ludwig 2008

Multifractal topological superconductor surface states

Spin singlet TSC: class CI [spin SU(2) symmetry]

- **Topologically-protected, gapless surface state (Bogoliubov) quasiparticles**
- **TRI disorder ➡ multifractal surface states (*n* replicas, 2 *k* valleys)**

Conformal Field Theory: $SO(4\,n\,k)_1 \supset Sp(2\,n)_k \oplus Sp(2\,k)_n$

- LDOS multifractal spectrum [$Sp(2\,n)_k$ primary fields]

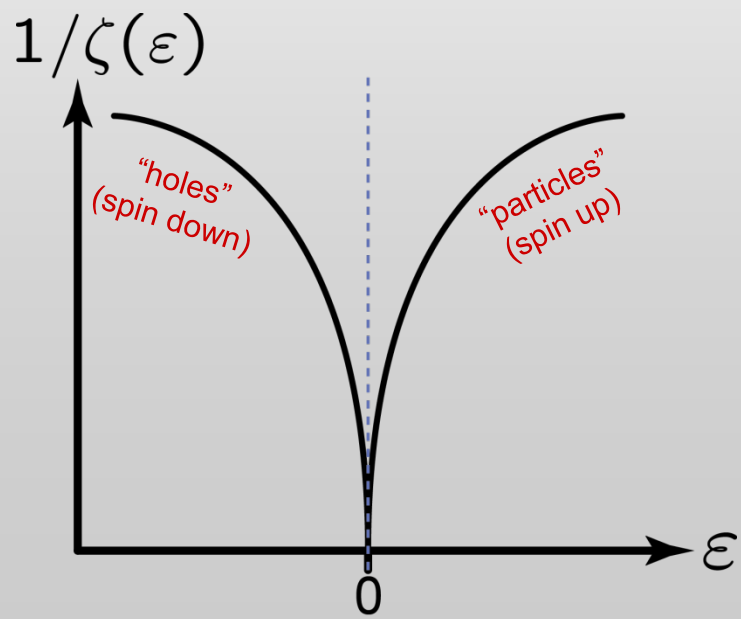
$$\tilde{\tau}(q) = 2(q-1) + \Delta_q - q\Delta_1 = (q-1)\left[2 - \frac{1}{2(k+1)}q\right]$$

***k = 1*:**
Nersesyan, Tsvelik, Wenger 1994
Mudry, Chamon, Wen 1996
Caux, Kogan, Tsvelik 1996

***k > 1*:**
Foster and Yuzbashyan 2012

Different from Z2 Topological insulators:

- Majorana surface band delocalized only at the chemical potential (which lies in bulk gap)
- At non-zero energy, crossover to orthogonal class



All other states localized! (~IQHP)

- Loc. length $\zeta(\varepsilon) \sim |\varepsilon|^{-1/z};\ z = \frac{4k+3}{2(k+1)}$

Generic surface model: Disorder and interactions

Add generic, *weak* **interparticle interactions**, consistent with bulk symmetries [time-reversal, spin SU(2), and valley symmetry*]

$$H_I = U \int d^2\mathbf{r}\, \Psi^\dagger_\alpha \Psi_\beta \Psi^\dagger_\gamma \Psi_\delta$$

Generic surface model: Disorder and interactions

Clean limit: DOS dimension $\Delta_1 = d - z$ determines relevance of short-ranged interactions

$$\frac{dU}{dl} = (\Delta_1 - \Delta_2^{(U)})U = -\Delta_1 U + \boldsymbol{O}\left(U^2\right), \quad \Delta_2^{(U)} = 2\Delta_1$$

Clean Dirac: $\Delta_1 = 1$ ➡ **interactions irrelevant!**

$$H_I = U \int d^2\mathbf{r}\, \Psi_\alpha^\dagger \Psi_\beta \Psi_\gamma^\dagger \Psi_\delta$$

Generic surface model: Disorder and interactions

Clean limit: DOS determines relevance of short-ranged interactions

$$\frac{dU}{dl} = (\Delta_1 - \Delta_2^{(U)})U = -\Delta_1 U + \boldsymbol{O}\left(U^2\right), \quad \Delta_2^{(U)} = 2\Delta_1$$

Clean Dirac: $\Delta_1 = 1$ ➡ **interactions irrelevant!**

Dirty case:

- $\Delta_1 \equiv$ scaling dimension of disorder-averaged LDOS
- $\Delta_2^{(U)} \equiv$ scaling dimension of disorder-averaged interaction

Constraint: $\Delta_2^{(U)} \geq \Delta_2$

- $\Delta_2 \equiv$ scaling dimension of second LDOS moment

Generic surface model: Disorder and interactions

Clean limit: DOS determines relevance of short-ranged interactions

$$\frac{dU}{dl} = (\Delta_1 - \Delta_2^{(U)})U = -\Delta_1 U + \boldsymbol{O}\left(U^2\right), \quad \Delta_2^{(U)} = 2\Delta_1$$

Clean Dirac: $\Delta_1 = 1$ ➡ **interactions irrelevant!**

Dirty case:

- $\Delta_1 \equiv$ scaling dimension of disorder-averaged LDOS
- $\Delta_2^{(U)} \equiv$ scaling dimension of disorder-averaged interaction

Under suitable circumstances $\Delta_2^{(U)} = \Delta_2$ (for particular interactions)

- $\Delta_2 \equiv$ scaling dimension of second LDOS moment

Convexity property for a multifractal extended surface state:

$\Delta_2 < 2\Delta_1$ **(independent dimensions!)** Duplantier and Ludwig 1991

$\therefore$ ***Wavefunction multifractality can amplify short-ranged interactions!***

Generic surface model: Disorder and interactions

For disordered TSC surface system, expect important interactions are

- Cooper pairing of surface quasiparticles (time-reversal invariance)
- Spin exchange (spin is conserved = hydrodynamic mode)

One particular combination turns out to be most important:

$$H_I = U \int d^2\mathbf{r} \left[m(\mathbf{r})\, m(\mathbf{r}) - 4\vec{S}(\mathbf{r}) \cdot \vec{S}(\mathbf{r}) \right] + \ldots (irrelevant)$$

Order parameters break time-reversal:

- **Spin polarization** $\vec{S} = c^\dagger \frac{\vec{\sigma}}{2} c$

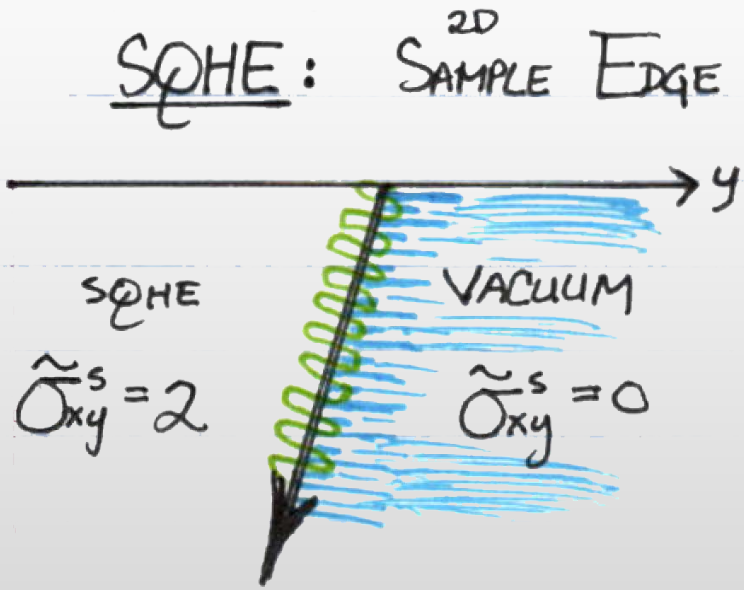
Class C
Spin QHE

- ***Imaginary* s-wave pairing mass** $m \sim -ic_\uparrow^\dagger c_\downarrow^\dagger + ic_\downarrow c_\uparrow; \quad \tilde{\sigma}_s^{xy} = k\,\mathrm{sgn}(m)$

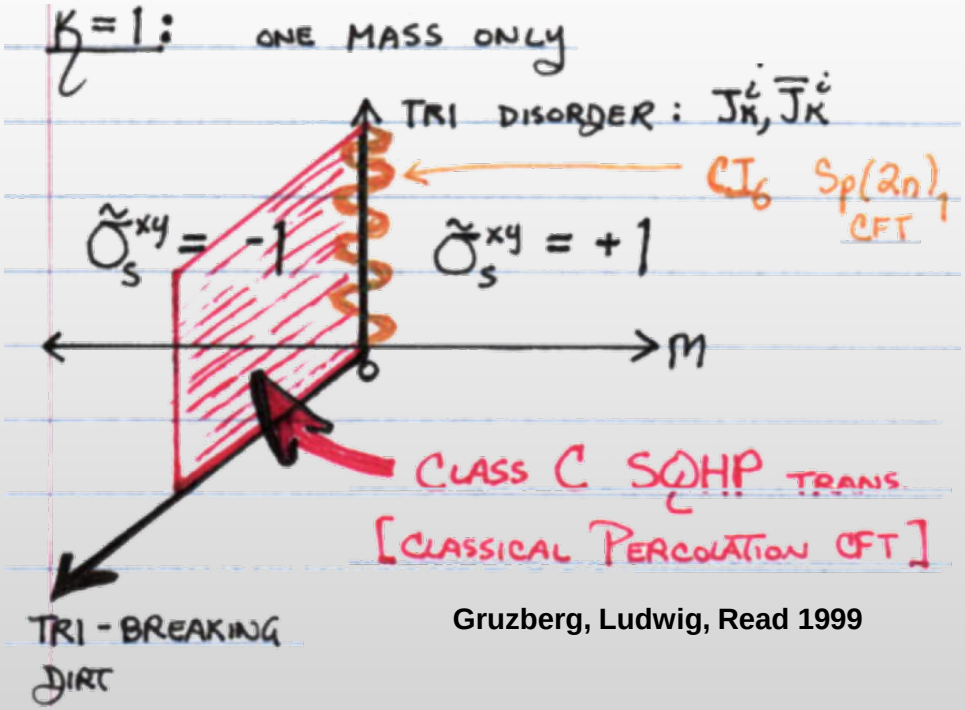
Aside: Topological "Half-(twice)-integer" Spin QHE

"Regular" 2D Spin QHE

Senthil, Marston, Fisher 1999



"Half"-Spin QHE (Top SC surface)



Gruzberg, Ludwig, Read 1999

Class C Spin QHE

- ***Imaginary* s-wave pairing mass** $m \sim -ic^\dagger_\uparrow c^\dagger_\downarrow + ic_\downarrow c_\uparrow; \quad \tilde{\sigma}^{xy}_s = k\,\mathrm{sgn}(m)$

Interaction mediated *instability* on TSC surface due to multifractality

$$H_I = U \int d^2\mathbf{r} \left[m(\mathbf{r})\, m(\mathbf{r}) - 4\vec{S}(\mathbf{r}) \cdot \vec{S}(\mathbf{r}) \right] + \ldots (irrelevant)$$

- LDOS multifractal spectrum [Sp(2 *n*)$_k$ primary fields]

$$\tilde{\tau}(q) = 2(q-1) + \Delta_q - q\Delta_1 = (q-1)\left[2 - \tfrac{1}{2(k+1)} q\right]$$

For *U* interaction, $\Delta_2^{(U)} = \Delta_2 = 0, \quad \Delta_1 = 1/2(k+1)$

$$\frac{dU}{dl} = (\Delta_1 - \Delta_2)U = \frac{1}{2(k+1)} U + O\left(U^2\right)$$ ***Relevant!***

Order parameters break time-reversal:

- **Spin polarization** $\vec{S} = c^\dagger \frac{\vec{\sigma}}{2} c$

Class C
Spin QHE

- ***Imaginary* s-wave pairing mass** $m \sim -ic_\uparrow^\dagger c_\downarrow^\dagger + ic_\downarrow c_\uparrow; \quad \tilde{\sigma}_s^{xy} = k\,\mathrm{sgn}(m)$

Interaction mediated *instability* on TSC surface due to multifractality

$$H_I = U \int d^2\mathbf{r} \left[m(\mathbf{r})\, m(\mathbf{r}) - 4\vec{S}(\mathbf{r}) \cdot \vec{S}(\mathbf{r}) \right] + \ldots$$

- LDOS multifractal spectrum [Sp(2 n)$_k$ primary fields]

$$\tilde{\tau}(q) = 2(q-1) + \Delta_q - q\Delta_1 = (q-1)\left[2 - \tfrac{1}{2(k+1)} q\right]$$

For *U* interaction, $\Delta_2^{(U)} = \Delta_2 = 0, \quad \Delta_1 = 1/2(k+1)$

$$\frac{dU}{dl} = (\Delta_1 - \Delta_2)U = \frac{1}{2(k+1)} U + O\left(U^2\right)$$

Physics:

Disorder induces multifractal enhancement in energy-resolved

- **spin** $\vec{S}(\varepsilon, \mathbf{r})$
- **pairing mass** $m(\varepsilon, \mathbf{r})$

"LDOS" fluctuations

Short-range LDOS fluctuations can enhance interaction effects

Order parameters break time-reversal:

- **Spin polarization** $\vec{S} = c^\dagger \frac{\vec{\sigma}}{2} c$
- ***Imaginary* s-wave pairing mass** $m \sim -ic_\uparrow^\dagger c_\downarrow^\dagger + ic_\downarrow c_\uparrow; \quad \tilde{\sigma}_s^{xy} = k\,\mathrm{sgn}(m)$

Class C
Spin QHE

Interaction mediated *instability* on TSC surface due to multifractality

$$H_I = U \int d^2\mathbf{r} \left[m^2(\mathbf{r}) - 4\vec{S} \cdot \vec{S}(\mathbf{r}) \right]$$

$$\frac{dU}{dl} = (\Delta_1 - \Delta_2)U = \frac{1}{2(k+1)} U$$

- **Due to disorder, *interactions are relevant!* (multifractal enhancement)**
- ***Expect time-reversal breaks spontaneously.***

$(\mathrm{A})\ U \to +\infty : \ \langle \vec{S} \rangle \neq 0$

- **(Local) ferromagnetic order**
- **Spin sym broken down to U(1)**
- **Unitary class A**
- **No gap in clean limit**
- **Anderson insulator with dirt (unless IQHP)**

$(\mathrm{B})\ U \to -\infty : \ \langle m \rangle \neq 0$

- **Imaginary s-wave pairing of surface QPs**
- **Spin SU(2) symmetry *preserved***
- **Gap opens in clean limit**
- **Insulating plateau of the Class C Spin QHE**

Order parameters break time-reversal:

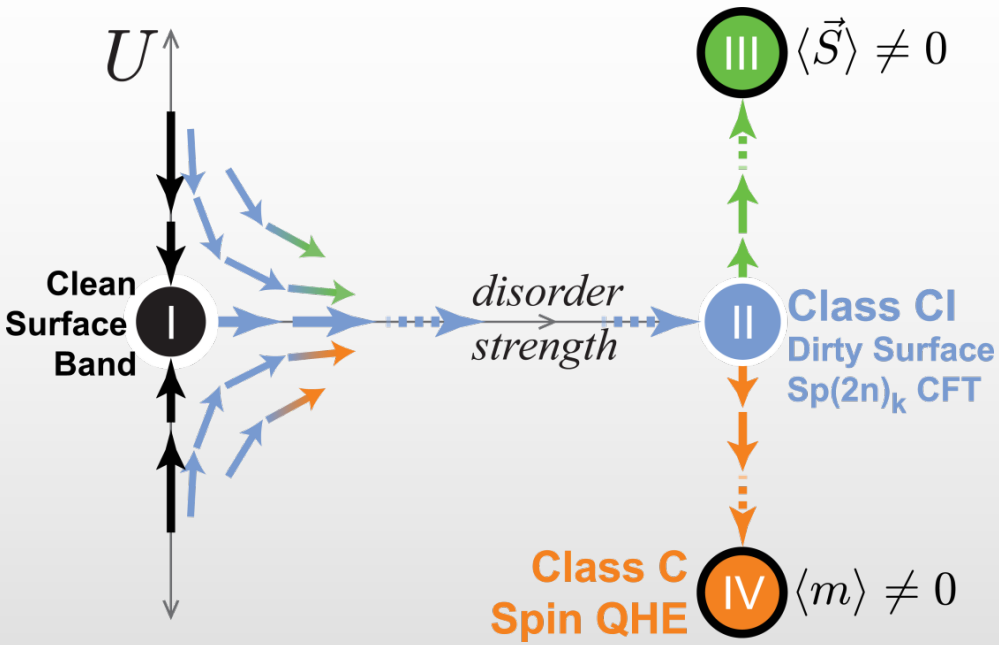
- **Spin polarization** $\vec{S} = c^\dagger \frac{\vec{\sigma}}{2} c$
- ***Imaginary* s-wave pairing mass** $m \sim -ic_\uparrow^\dagger c_\downarrow^\dagger + ic_\downarrow c_\uparrow; \quad \tilde{\sigma}_s^{xy} = k\,\mathrm{sgn}(m)$

Class C Spin QHE

Interaction mediated *instability* on TSC surface due to multifractality

$$H_I = U \int d^2\mathbf{r} \left[m^2(\mathbf{r}) - 4\vec{S} \cdot \vec{S}(\mathbf{r}) \right]$$

$$\frac{dU}{dl} = (\Delta_1 - \Delta_2)U = \frac{1}{2(k+1)} U$$



- **Due to disorder, *interactions are relevant!*** (multifractal enhancement)
- ***Expect time-reversal breaks spontaneously.***

- ***Gapless Andreev states for the class CI 3D TSC* (probably) *do not exist*.**

$\therefore$ ***Interactions plus disorder can break surface topological protection.***

Foster and Yuzbashyan 2012

Check: Many valleys, sigma model

Key tools: non-abelian bosonization, conformal embedding

Conformal Field Theory: $SO(4\,n\,k)_1 \supset Sp(2\,n)_k \oplus Sp(2\,k)_n$

"The normal life": Diagrammatics

1. **Self-energy rainbows (SCBA) → Finite (non-zero, non-divergent) DOS**
2. **Diffusons and Cooperons via ladders (Gaussian fluctuations)**
3. **Hikami box (1 loop), higher order corrections**

[non-linear sigma model (NLsM) Wegner 1979]

4. **Add interactions (Altshuler and Aronov 1979, Finkelstein 1984)**

Check: Many valleys, sigma model

Key tools: non-abelian bosonization, conformal embedding

Conformal Field Theory: $SO(4\,n\,k)_1 \supset Sp(2\,n)_k \oplus Sp(2\,k)_n$

"The normal life": Diagrammatics

1. **Self-energy rainbows (SCBA) → Finite (non-zero, non-divergent) DOS**
2. **Diffusons and Cooperons via ladders (Gaussian fluctuations)**
3. **Hikami box (1 loop), higher order corrections**

[non-linear sigma model (NLsM) Wegner 1979]

4. **Add interactions (Altshuler and Aronov 1979, Finkelstein 1984)**

For 2D Dirac fermions, program fails at step 1.

Nersesyan, Tsvelik, Wenger 1994; Aleiner and Efetov 2006

Integrating-out 2D Dirac fermions does not give controlled derivation of NLsM:

- **can miss WZW/topological terms**
- **NLsM in strongly-coupled regime**

Check: Many valleys, sigma model

Key tools: non-abelian bosonization, conformal embedding

Conformal Field Theory: $SO(4\,n\,k)_1 \supset Sp(2\,n)_k \oplus Sp(2\,k)_n$

Alternative path to the sigma model: C.f. Altland, Simons, Zirnbauer 2002

1. **Bosonize, embed, fractionalize: $SO(4\,n\,k)_1 \Rightarrow Sp(2\,n)_k$**
2. **Bosonize: $Sp(2\,n)_k$ = principal chiral NLsM plus WZW term**
3. **Take large-k limit (many valleys)**
   - **(Zero Temp, Landauer) Conductance $G \sim k\,e^2/h$** ***weak coupling!***
   - **Neglect WZW term (unimportant for large *k* "classical" limit)**
   - **Incorporate interactions for hydrodynamic modes**

Check: Many valleys, sigma model

Interacting (Finkel'stein) NLsM for class CI

$$S = \frac{1}{2\lambda}\int_{\mathbf{r}} \mathrm{Tr}\left[\boldsymbol{\nabla}\hat{Q}^\dagger \cdot \boldsymbol{\nabla}\hat{Q}\right] - h\int_{\mathbf{r}} \mathrm{Tr}\left[|\hat{\omega}_N|\hat{\Sigma}^3\left(i\hat{Q} - i\hat{Q}^\dagger\right)\right] + k\Gamma_{\mathrm{WZW}}$$

$$+\Gamma_t \sum_a \int_{\tau,\mathbf{r}} \left(\mathrm{Tr}_s\left\{\hat{\vec{S}}\left[\hat{Q}_{aa}(\tau,\tau) - \hat{Q}^\dagger_{aa}(\tau,\tau)\right]\right\}\right)^2$$

$$+\Gamma_c \sum_a \int_{\tau,\mathbf{r}} \left\{\mathrm{Tr}_s\left[\hat{Q}_{aa}(\tau,\tau) + \hat{Q}^\dagger_{aa}(\tau,\tau)\right]\right\}^2$$

- **"Resistance"** $\lambda = 8\pi/k$
- **Interaction couplings:**

  1. Repulsive spin-triplet exchange: $\Gamma_t > 0$

  2. Repulsive Cooper channel: $\Gamma_c > 0$

Check: Many valleys, sigma model

1 loop RG: **Dell'Anna (2006) (ignoring WZW term)**

$$\frac{d\gamma_t}{dl} = -\frac{\lambda}{2}\gamma_c\left(1-\gamma_t\right)\left(1-2\gamma_t\right)$$

$$\frac{d\gamma_c}{dl} = \frac{\lambda}{2}\left\{-3\gamma_t - 2\gamma_c + 3\gamma_c\left[\log(1-\gamma_t)+\gamma_t\right]\right\} - \gamma_c^2$$

Linearize: $\frac{d\ln\gamma_{\mathrm{r}}}{dl} = \frac{\lambda}{2}, \quad \frac{d\ln\gamma_{\mathrm{i}}}{dl} = -\frac{3\lambda}{2}, \quad \gamma_{\mathrm{r}} \equiv \gamma_c - 3\gamma_t, \ \gamma_{\mathrm{i}} \equiv \gamma_c + \gamma_t$

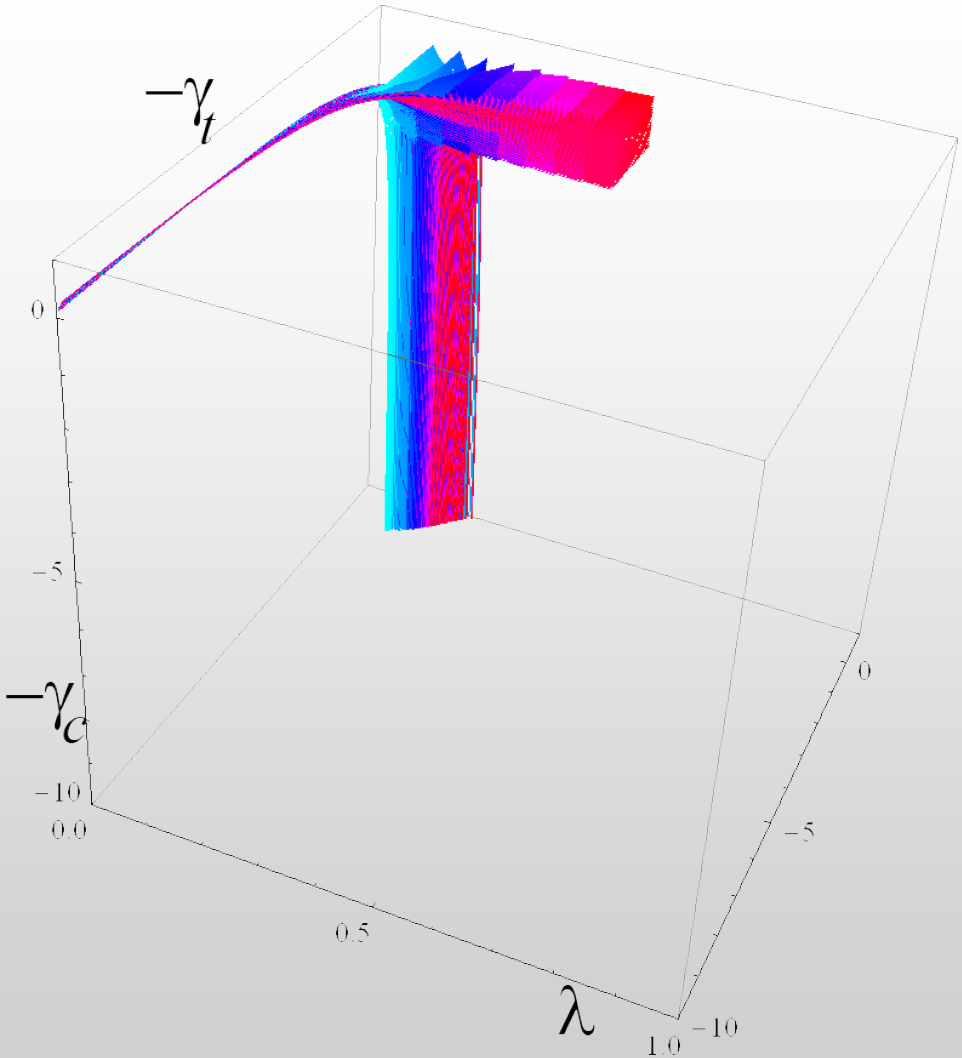
CFT: **In k $\rightarrow \infty$ limit, two interactions become marginal**

$$H_I = \int d^2\mathbf{r}\left\{U\left[m^2(\mathbf{r}) - 4\vec{S}\cdot\vec{S}(\mathbf{r})\right] + W\mathcal{O}_W\right\}$$

$$\frac{d\ln U}{dl} = \frac{1}{2(k+1)}, \quad \frac{d\ln W}{dl} = -\frac{3}{2(k+1)}$$

- ***Difference of repulsive Cooper pairing, spin exchange interactions relevant***
- ***Using full 1-loop RG, metal is unstable to either infinite spin exchange (Stoner $\gamma_t \rightarrow -\infty$) or attractive Cooper pairing ($\gamma_c \rightarrow -\infty$)***

Hybrid FNLsM + WZW RG: Unstable surface states



Including feedback on resistance λ does not change the outcome:

Dell'Anna 2006, Foster unpublished

$$\frac{d\lambda}{dl} = \lambda^2\left[1-(\lambda k)^2\right] + \lambda^2\left\{3\left[1+\left(\frac{1-\gamma_t}{\gamma_t}\right)\ln\left(1-\gamma_t\right)\right]-\frac{\gamma_c}{2}\right\}$$

Summary and outlook

- **New avenues for Anderson localization physics: local probes and LDOS fluctuations on the surface of topological insulators and superconductors**

- **Combined effects of disorder plus interparticle interactions can destroy topologically-protected surface states**

- **2 other TSC classes in 3D with WZW-CFT surface states:**

  a) Class DIII: no spin symmetry, TRI (e.g., ^{3}He B, $CuBi_2Se_3$?)

  b) Class AIII: spin U(1) symmetry, TRI (e.g., spin-triplet p-wave)

DIII: No spin symmetry (i.e. SOC)

a) Class DIII: no spin symmetry, TRI (e.g., ^{3}He B, $CuBi_2Se_3$?)

- **k = 1 valley (^{3}He B): No disorder! Stable.**
- **k = 2 valleys: Same as AIII k = 1 (Abelian VP). Stable.**
- ***k* > 2 valleys: $SO(n\,k)_1 \supset SO(n)_k \oplus SO(k)_n$**
- **WZW CFT is *already unstable* in absence of interactions**
- **Flow to weakly-coupled diffusive metal**

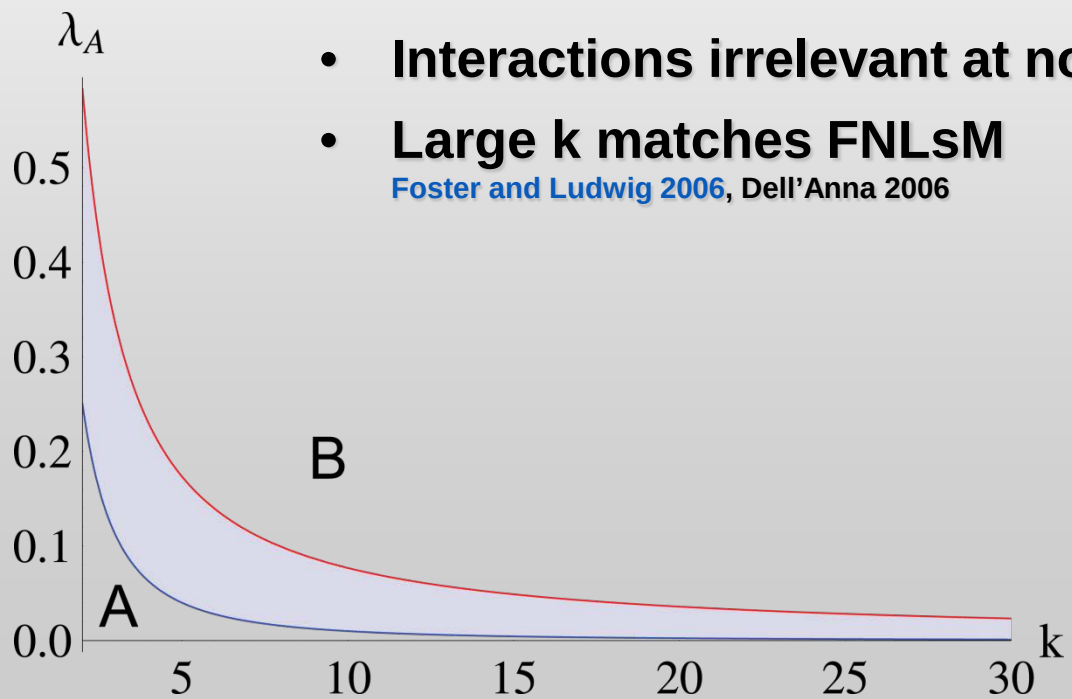
$$\frac{d\lambda}{dl} = -2\lambda^2\left[1-(k\lambda)^2\right]$$

- **FNLsM: Pairing interactions only (spin not hydrodynamic)**
- **Expect usual BCS instability for attractive pairing potential (Anderson's theorem)**
  Surface pairing gap: *imaginary pairing* (broken TRI)

AIII: Spin U(1) symmetry

b) Class AIII: spin U(1) symmetry, TRI (e.g., spin-triplet p-wave)

- **k = 1 valley (Abelian VP): Stable for weak disorder, interactions**
- ***k* > 1 valleys: $U(1) \oplus SU(n\,k)_1 \supset U(1) \oplus SU(n)_k \oplus SU(k)_n$**
- **Abelian disorder parameter λ_A (z-spin current disorder)**
- **Triplet exchange, singlet pairing interactions**



- **Interactions irrelevant at non-zero λ_A**
- **Large k matches FNLsM**
  Foster and Ludwig 2006, Dell'Anna 2006

Foster (unpublished)

AIII: Spin U(1) symmetry

b) Class AIII: spin U(1) symmetry, TRI (e.g., spin-triplet p-wave)

- **k = 1 valley (Abelian VP): Stable for weak disorder, interactions**
- ***k* > 1 valleys: U(1) ⊕ SU($n\,k$)$_1$ ⊃ U(1) ⊕ SU(n)$_k$ ⊕ SU(k)$_n$**
- **Abelian disorder parameter λ_A (z-spin current disorder)**
- **Triplet exchange, singlet pairing interactions**
- **<u>Feedback effect</u>: generic instability of non-interacting surface**

$$\frac{d\lambda}{dl} = \lambda^2 \left(2 \left[1 + \left(\frac{1-\gamma_t}{\gamma_t} \right) \ln\left(1-\gamma_t\right) \right] - \gamma_c \right)$$

$$\frac{d\lambda_A}{dl} = \lambda^2 \left[1 - (k\lambda)^2 \right] + 2\lambda_A \frac{d\ln\lambda}{dl}$$

Guruswamy, LeClair, and Ludwig 2000,
Foster and Ludwig 2006, Dell'Anna 2006

Foster
(unpublished)